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Newton polyhedra of discriminants: A computation. (English) Zbl 1101.14041

Lossen, Christoph (ed.) et al., Singularities and computer algebra. Selected papers of the conference, Kaiserslautern, Germany, October 18–20, 2004 on the occasion of Gert-Martin Greuel's 60th birthday. Cambridge: Cambridge University Press (ISBN 0-521-68309-2/pbk). London Mathematical Society Lecture Note Series 324, 185-210 (2006).

The paper deals with the non-constancy of the Jacobian Newton polygon in an equisingular family of complete intersection branches. In more details, for a hypersurface germ $f(u_0..u_n) = 0$ consider the map $(l, f) : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}^2, 0)$. Here l is a generic linear form. Let t_0, t_1 be the coordinates on $\mathbb{C}^2$. The Jacobian Newton polygon of the hypersurface is defined to be the Newton polygon of the discriminant of the above map (in the coordinates (t_0, t_1)). In general, under equisingular deformations of the hypersurface $(f = 0)$, the discriminant changes (e.g. the number of branches can vary). However the Newton polygon of the discriminant is known to be constant. Even more, in case of uni-branched plane curves the Jacobian Newton polygon is a complete invariant of the singularity type (e.g., the semi-group or Puiseux characteristics can be restored). A natural question therefore is the constancy of the Jacobian Newton polygon for curves that are locally complete intersections.

The paper answers this question negatively by an explicit counterexample. The authors consider degeneration of a plane analytic branch to a monomial curve (preserving the semi-group). The Jacobian Newton polygon in this case is not constant. However, for the given counterexample the information contained in the Jacobian Newton polygon (i.e. the semigroup) remains constant and only the encoding changes. The paper is of explicit computational character.

For the entire collection see [Zbl 1086.14001].

Reviewer: Dmitry Kerner (Bonn)

MSC:

14H20 Singularities of curves, local rings
32S10 Invariants of analytic local rings
32S15 Equisingularity (topological and analytic)

Keywords:

Newton polygon; discriminant; polar hypersurface