Csörgő, Piroska; **Kepka, Tomáš**
On loops whose inner permutations commute. (English) Zbl 1101.20034
Commentat. Math. Univ. Carol. 45, No. 2, 213-221 (2004).

Let Q be a loop, $G = M(Q)$ its multiplication group, and $H = I(Q)$ its inner mapping group. The paper studies certain aspects of G under the assumption that H is Abelian.

Denote by K the normal closure of H in G. Then $K = G'H$, for every loop (Proposition 3.1). If H is Abelian, then $Z(G) \cap K \neq 1$, $Z(K) \neq 1$ and $Z(K)H \neq H$ (Proposition 3.6). In Section 4 one assumes, in addition, the existence of $P \leq Z(G) \cap K$ with $PH \trianglelefteq K$. This gives a number of consequences; for example that $G''' = 1$ and that K is nilpotent of class at most two. In the last section one proves, amongst others, that if Q is nilpotent of class at least three, then every prime dividing $|H|$ divides $|Q|$.

The paper uses the language of H-connected transversals. That makes it look very formal. However, when a translation is made into the language of structural loop theory, the statements usually acquire a clear meaning. In a few cases one even discovers that rather obvious facts are being proved – for example Lemma 2.10 is a veiled form of saying that a loop is a group if and only if the left and the right translations commute.

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MSC:

20N05	Loops, quasigroups
20D10	Finite solvable groups, theory of formations, Schunck classes, Fitting classes, π-length, ranks
20D40	Products of subgroups of abstract finite groups

Keywords:

loops; connected transversals; Abelian inner mapping groups; multiplication groups

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