

Bourbaki, Nicolas

Elements of mathematics. Commutative algebra. Chapters 5–7. Reprint of the 1985 original. (Éléments de mathématique. Algèbre commutative. Chapitres 5 à 7.) (French) [Zbl 1103.13002](#)
Berlin: Springer (ISBN 3-540-33941-8/pbk). 351 p. (2006).

This volume comprises the unaltered reprint of Chapters 5–7 of the original French edition of N. Bourbaki’s treatise “Commutative Algebra”. Chapters 5 and 6 were first published and reviewed in *N. Bourbaki* [(1964; [Zbl 0205.34302](#))], whereas Chapter 7 followed in (1965; [Zbl 0141.03501](#)). Now as before, these chapters are devoted to the following fundamental concepts in commutative algebra:

Chapter 5 treats integral ring extensions and algebraic integers. This includes integrally closed domains, integers over a graded ring, lifting and ramification properties of prime ideals, and the basic theory of finitely generated algebras over a field. The latter topic contains, among other things, the Noether normalization lemma, Hilbert’s Nullstellensatz, and a discussion of Jacobson rings.

Chapter 6 provides an extensive account of valuation theory for rings and fields, together with its topological aspects (completions and locally compact fields) and the related ramification theory. Chapter 7 develops the general algebraic theory of divisors, with particular emphasis put on Krull rings, Dedekind rings, factorial rings, and their respective module theory. As it is usual for Bourbaki’s treatises, each section comes with a large number of supplementing exercises, many of which point to further results and developments of the theory.

Reviewer: [Werner Kleinert \(Berlin\)](#)

MSC:

- [13–01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
- [13A18](#) Valuations and their generalizations for commutative rings
- [13B22](#) Integral closure of commutative rings and ideals
- [13B35](#) Completion of commutative rings
- [13C20](#) Class groups
- [13F05](#) Dedekind, Prüfer, Krull and Mori rings and their generalizations

Cited in 3 Reviews Cited in 15 Documents

Keywords:

[textbook](#); [commutative rings](#); [ring extensions](#); [valuations](#); [topological rings and fields](#); [algebraic integers](#); [finitely generated algebras](#)