

**Bourbaki, Nicolas**

**Elements of mathematics. Commutative algebra. Chapters 8 and 9. Reprint of the 1983 original.** (*Éléments de mathématique. Algèbre commutative. Chapitres 8 et 9.*) (French)

Zbl 1103.13003

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This third volume of N. Bourbaki's "Commutative Algebra" delivers the remaining Chapters 8 and 9, which were published almost twenty years after the first appearance of the foregoing Chapters 1–7. Apparently elaborated by another generation of Bourbaki members, these two supplementary chapters were made public in (1983; Zbl 0579.13001). Unlike the foregoing chapters, this part of Bourbaki's treatise has neither been reprinted ever since, nor has it undergone a translation into English. Having been out of print for more than a decade, this isolated part of Bourbaki's treatise "Commutative Algebra" has now become an integral part of the present new edition in three volumes, very much so for the benefit of the entire mathematical community. As it ought to be expected, these two last chapters turn to some more advanced (and more specific) topics in commutative algebra, chiefly to such concepts that proved to be crucial in the more recent developments of modern algebraic and arithmetic geometry. Chapter 8 treats the diverse notions of dimension in the theory of rings, algebras, and modules. In this context, Hilbert-Samuel series for graded (or filtered) modules over polynomial rings, the structure of regular local rings, and multiplicity theories for modules over noetherian rings are also presented.

Chapter 9 is more specific in that it develops the advanced structure theory for complete local noetherian rings. This includes a description of the ring of Witt vectors in characteristic  $p > 0$ , an analysis of Cohen rings, a treatment of  $p$ -rings, and the study of the integral closure of a complete local ring by means of Japanese rings and Nagata rings. This material is very fundamental with a view to the framework of Grothendieck's formal algebraic geometry, on the one hand, and to modern algebraic number theory, on the other hand.

According to the typical Bourbaki style, much more material and various concrete examples are touched upon in the numerous exercises attached to each section of these two chapters. Also, there are three short appendices complementing the material on local rings and  $p$ -rings, which come with their own set of related exercises. In contrast to the foregoing chapters 1–7, the exercises to Chapters 8 and 9 not only seem to be more plentiful, but also appear with more detailed hints and instructions. However, in return for that, there are much fewer historical remarks than in the volumes elaborated by the older Bourbaki generation. In this regard, the original Bourbaki style appears to have slightly changed, although the overall approach to mathematics has been maintained in the traditional manner.

Reviewer: [Werner Kleinert \(Berlin\)](#)

**MSC:**

- 13-02 Research exposition (monographs, survey articles) pertaining to commutative algebra
- 13C15 Dimension theory, depth, related commutative rings (catenary, etc.)
- 13J10 Complete rings, completion
- 13K05 Witt vectors and related rings (MSC2000)
- 13H05 Regular local rings
- 13H15 Multiplicity theory and related topics
- 13E05 Commutative Noetherian rings and modules

Cited in **2** Reviews  
Cited in **18** Documents

**Keywords:**

[research monographs](#); [commutative rings and algebras](#); [dimension theory](#); [complete rings](#); [multiplicity theory](#); [Witt vectors](#); [regular local rings](#); [Japanese rings](#); [Nagata rings](#)

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