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Conditional and unconditional well-posedness for nonlinear evolution equations. (English)

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Summary: Attention is given to the question of well-posedness in Hadamard's classical sense for nonlinear evolution equations of the form

$$\frac{du}{dt} + Lu = N(u), \quad u(0) = \varphi. \quad (1)$$

In view are various classes of nonlinear wave equations, nonlinear Schrödinger equations, and the (generalized) KdV equations. Equations (1) are often well posed in a scale X_s , say, of Banach spaces, at least for s large enough. Here, increasing values of s correspond to more regularity; thus $X_r \subset X_s$ if $r > s$. For smaller values of s , some equations of the form (1) are well-posed in a conditional sense that the uniqueness aspect depends upon the imposition of auxiliary conditions. In the latter context, it is natural to inquire whether or not the auxiliary conditions are essential to securing uniqueness.

It is shown here that for a conditionally well-posed Cauchy problem (1), the auxiliary specification is removable if a certain persistence of regularity holds. As a consequence, it will transpire that a conditionally well-posed problem (1) is (unconditionally) well-posed if the aforementioned persistence property holds.

These results are applied to study several recent conditional well-posedness results for the KdV equation, nonlinear Schrödinger equations, and nonlinear wave equations. It is demonstrated that the auxiliary conditions used to secure the uniqueness are all removable and the corresponding Cauchy problems are, in fact, unconditionally well-posed as long as their classical solutions exist globally. In addition, the well-posedness for an initial-boundary-value problem for the KdV equation posed in a quarter plane is also considered. An affirmative answer is provided for a uniqueness question left open in a recent paper of *J. E. Colliander* and *C. E. Kenig* [Commun. Partial Diff. Equ. 27, 2187–2266 (2002; Zbl 1041.35064)].

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)

35Q53 KdV equations (Korteweg-de Vries equations)

35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs

34G20 Nonlinear differential equations in abstract spaces

35K90 Abstract parabolic equations

35L90 Abstract hyperbolic equations

Cited in 14 Documents

Keywords:

well-posedness; nonlinear Schrödinger equations; KdV equations; uniqueness; Cauchy problem