Bourbaki, Nicolas

Elements of mathematics. Algebra. Chapter 10. Homological algebra. Reprint of the 1980 original. (Éléments de mathématique. Algèbre. Chapitre 10. Algèbre homologique.) (French)

Zbl 1105.18001
Berlin: Springer (ISBN 3-540-34492-6/pbk). 216 p. (2007).

N. Bourbaki's monumental opus "Éléments de mathématique", initiated by the founders of this formerly secret group of French mathematicians in the mid-1930s and aiming at a profound systematization of various areas of modern mathematics, grew in the following 65 years into a totality of ten fundamental "Books", with over sixty chapters altogether. Each book was devoted to a particular basic discipline in current pure mathematics, and some of the constituting (later) chapters of some of the books were added in remarkably varying time intervals, sometimes obviously elaborated by different generations of Bourbaki group members.

Book II of Bourbaki's "Éléments de mathématique" is titled "Algèbre" and consists of ten chapters, with the so far last chapter (Chapter 10) having been published in 1980, that is more than twenty years after the appearance of the that far final Chapter 9.

The book under review is the faithful, unabridged reprinting of the 1980 French original of Chapter 10 of Bourbaki's "Algèbre", which back then appeared with the subtitle "Algèbre homologique" (1980; Zbl 0455.18010).

In fact, the present reproduction is actually the original French book, formerly published by Masson, Paris, with just an additional (yellow) cover attached by Springer Verlag. Anyway, in this fashion, the apparently rather limited original edition of Bourbaki's "Algèbre homologique" has finally become available again, after having been out of print for nearly twenty years, thanks to Springer's splendid reprinting project with regard to all Bourbaki volumes in their entirety.

As to the contents of the present chapter on basic homological algebra, which apparently has never gained the same degree of popularity and circulation as the classical early chapters of Bourbaki's "Algèbre" or "Algèbre commutative", we just recall that it is composed of nine sections treating the following topics:

1. Complements of linear algebra (Commutative diagrams of module homomorphisms, flat modules, modules of finite representation, injective modules, injective hulls, and structure theory of injective modules);

2. Complexes of A-modules (Operations on complexes, the connecting homomorphism and the long homology sequence, homotopy of complexes, mapping cones and mapping cylinders of morphisms of complexes, Euler-Poincaré characteristics, complexes of bimodules, and the de Rham complex);

3. Resolutions (Free resolutions, injective resolutions, minimal projective resolutions, graded resolutions, the standard resolution, and the Grothendieck group);

4. Torsion products (Tensor products of complexes of modules, torsion products of modules and $\mathrm{Tor}_n^A(M,N)$, the exact Tor-sequence, and Künneth formulae);

5. Extension modules (Complexes of homomorphisms, extension modules and $\mathrm{Ext}_A^n(M,N)$, the exact Ext-sequence, characterization of projective and injective modules via extension modules, universal coefficent theorems, and generalizations to multi-modules);

6. Using non-canonical resolutions (Computational aspects of Ext and Tor, finiteness theorems, canonical standard homomorphisms for Ext and Tor, and – as an application – the homology and the cohomology of groups);

7. Composition products (Operations on Ext and Tor, pairings between Ext and Tor, the Yoneda characterization of Ext, and further canonical connecting homomorphisms);

8. Homological dimension (Projective dimension of a module, homological dimension of a ring, rings of homological dimension 0 and 1, homological dimension of polynomial rings, Hilbert's Theorem of Syzygies, and homological dimension of graded modules);

9. J. Koszul complexes (Definition and properties of Koszul complexes, regular M-sequences via Koszul

complexes, extension classes associated to regular sequences, and various examples and applications).

At the end of the book, there is the obligatory, immense collection of further-leading exercises, grouped with respect to the different sections of this chapter, as it is notorious for the classical Bourbaki style. Alas, another outstanding feature of the earlier Bourbaki treatises, namely the familiar supplement of respective historical notes, presumably inspired by Bourbaki's former disciplinarian, Jean Dieudonné, is not found in this volume. Nevertheless, the rich collection of about 160 exercises absolutely follows the good old Bourbaki tradition in regard of their comprehensiveness, versality, systematic representation, degree of difficulty, and propelling character. In fact, various topics not explicitely touched upon in the text, as for example regular local rings, spectral sequences, group cohomology, Galois cohomology, Lie algebra cohomology, simplicial schemes, coherent modules, and many other related concepts, are subject to extended exercises, with concrete hints for solution amply provided.

On the other hand, many current topics of homological algebra do not occur in the present booklet of 216 pages. First of all, homological algebra is here restricted to the category of modules over a ring, and any categorical or functorial aspects of general homological algebra are consequently left out, including derived functors, adjoint functors, satellites, and other powerful standard constructions of conceptual significance. Well, Bourbaki remained true to their traditional, often criticized conception of not integrating the categorical framework in their systematic representation, which is no particular surprise. On the other hand, there are meanwhile numerous excellent textbooks on general homological algebra for further reading, which comfortably compensate this drawback of Bourbaki's text, and which can be used to solve many of Bourbaki's tough exercises, in a very educating manner. The main feature of Bourbaki's text on homological algebra is its fairly elementary, self-contained and very detailed exposition, besides its traditional methodological compactness.

Albeit the presentation is (just as traditionally) highly technical, functional, and barely motivated, Bourbaki's "Algèbre homologique" is and remains one of the (less widespread) classics in the field, and an utmost valuable reference book for further generations at any rate.

Reviewer: Werner Kleinert (Berlin)

MSC:

18-02	Research exposition (monographs, survey articles) pertaining to category theory
18Gxx	Homological algebra in category theory, derived categories and functors
00A05	Mathematics in general
01A72	Schools of mathematics
01A75	Collected or selected works; reprintings or translations of classics

Keywords:

homological algebra; research monograph; reference book; general mathematics; homology theory; homological dimension; Koszul complex; cohomology of groups

Full Text: DOI