Kawohl, B.; Fridman, V.

Isoperimetric estimates for the first eigenvalue of the p-Laplace operator and the Cheeger constant. (English) Zbl 1105.35029

Commentat. Math. Univ. Carol. 44, No. 4, 659-667 (2003).

The eigenvalue $\lambda = \lambda_p(\Omega)$ corresponding to a positive weak solution $u \in W_0^{1,p}(\Omega)$ of the homogeneous Dirichlet boundary-value problem for the equation $\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda |u|^{p-2}u = 0$, characterized by the Rayleigh quotient $\lambda_p(\Omega) = \min_{v \in W_0^{1,p}(\Omega)} \int_\Omega |\nabla v|^p \,dx / \int_\Omega |v|^p \,dx$, is addressed and related with Cheeger's constant $h(\Omega)$ defined as the infimum of $\operatorname{meas}_{n-1}(\partial D)/\operatorname{meas}_n(D)$ over all smooth subdomains $D \subset \Omega \subset \mathbb{R}^n$ whose boundary ∂D does not touch $\partial\Omega$. A subset $D \subset \Omega$ with $\operatorname{meas}_{n-1}(\partial D)/\operatorname{meas}_n(D) = h(\Omega)$ is called a Cheeger domain. It is proved that $\lambda_p(\Omega) \to h(\Omega)$ for $p \to 1$, and the corresponding eigenfunction u_p (if normalized), converges to the characteristic function of the Cheeger set. It implies, in particular, that the Cheeger set is convex if Ω is convex.

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MSC:

35J20	Variational methods for second-order elliptic equations
35J70	Degenerate elliptic equations
49R50	Variational methods for eigenvalues of operators (MSC2000)

Keywords:

Faber-Krahn type inequality; eigenvalue for p-Laplacian; Cheeger set; 1-Laplace operator

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