Lackey, Brad

On the Gauss-Bonnet-Chern theorem in Finsler geometry. (English) Zbl 1105.53055

Antonelli, Peter L. (ed.), Handbook of Finsler geometry. Vols. 1 and 2. Dordrecht: Kluwer Academic Publishers (ISBN 1-4020-1557-7/set; 1-4020-1555-0/v.1; 1-4020-1556-9/v.2). 491-509 (2003).

Studying of the Gauss-Bonnet formula is one of the most important central topics in modern differential geometry. This formula gives the relationship between curvature and "angular excess". The modern view of the Gauss-Bonnet formula is that the curvature of a Riemannian manifold reflects the topology of the space. In 1944 *S. S. Chern* [Ann. Math. 45, 747–752 (1944; Zbl 0060.38103)] proved the Gauss-Bonnet formula of *Allendoerfer* and *Weil* [Trans. Amer. Math. Soc. 53, 101–129 (1943; Zbl 0060.38102)] using of the so-called "method of transgression". The method of transgression has set the standard for the generalisation of the Gauss-Bonnet formula to Finsler manifolds. At first *A. Lichnerowicz* [Comment. Math. Helv. 22, 271–301 (1949; Zbl 0039.17501)] extended the Gauss-Bonnet formula to a very restricted class of Finsler spaces (to Berwald spaces) by applying the method of transgression to the Pfaffian of the Cartan curvature. *D. Bao* and *S. S. Chern* [Ann. Math. 143, 233–252 (1996; Zbl 0849.53046)] discovered that the Chern connection is better for proving the Gauss-Bonnet formula.

B. Lackey, the author of this Part 5 of the Handbook of Finsler Geometry, showed a simple modification to the technique of Bao and Chern. Normalising by the typically non-constant volume of the fibre before applying the method of transgression, Lackey found a correction term that indeed sums with the normalised Pfaffian of the Chern curvature form to represent the Euler class for any Finsler space [Bull. Lond. Math. Soc. 34, 329–340 (2002; Zbl 1039.53083)].

For the entire collection see [Zbl 1057.53001].

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MSC:

53C60 Global differential geometry of Finsler spaces and generalizations (areal metrics)

Keywords:

Gauss-Bonnet theorem