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Extremal problems for transversals in graphs with bounded degree. (English) [Zbl 1106.05100]

Summary: We introduce and discuss generalizations of the problem of independent transversals. Given a
graph property $R$, we investigate whether any graph of maximum degree at most $d$ with a vertex partition
into classes of size at least $p$ admits a transversal having property $R$. In this paper we study this problem
for the following properties $R$: “acyclic”, “$H$-free”, and “having connected components of order at most $r$”.
Zbl 1033.05083)]. We prove that if the vertex set of a $d$-regular graph is partitioned into classes of size $d + \lfloor d/r \rfloor$, then it is possible to select a transversal inducing vertex disjoint trees on at most $r$ vertices.
Our approach applies appropriate triangulations of the simplex and Sperner’s lemma. We also establish
some limitations on the power of this topological method.

We give constructions of vertex-partitioned graphs admitting no independent transversals that partially
settles an old question of B. Bollobás, P. Erdős and E. Szemerédi [Discrete Math. 13, 97-107 (1975;
Zbl 0306.05121)]. An extension of this construction provides vertex-partitioned graphs with small degree such
that every transversal contains a fixed graph $H$ as a subgraph.

Finally, we pose several open questions.

MSC:

- 05D15 Transversal (matching) theory
- 05C35 Extremal problems in graph theory
- 05C69 Vertex subsets with special properties (dominating sets, independent
  sets, cliques, etc.)

Keywords:

- independent transversals; extremal problems; graphs with bounded degree

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