

Bourbaki, Nicolas

Elements of mathematics. Algebra. Chapter 9. Reprint of the 1959 original. (Éléments de mathématique. Algèbre. Chapitre 9.) (French) [Zbl 1107.13001](#)
Berlin: Springer (ISBN 3-540-35338-0/pbk). 211 p. (2007).

N. Bourbaki's fundamental work "Elements of Mathematics" is divided into several books and each book into several chapters. Book II in this series is titled "Algebra" and comprises 10 chapters, out of which Chapters 1 to 7 also have been translated into English (by P. M. Cohn and J. Howie). This English translation was published by Springer Verlag in 1980, and that in two volumes (Chapters 1–3 and Chapters 4–7). Chapters 8, 9 and 10 appeared separately and successively between 1959 and 1980, and so far only their French originals have been available, irrespective of their immediate translations into Russian. Having been out of print for a long time, also these three later chapters of Bourbaki's "Algebra" have finally become available, again, thanks to a recent reprinting program launched by Springer Verlag.

The booklet under review is the faithful reproduction of the French original edition of Chapter 9 of Book II, which was first published in 1959 by Hermann, Editeurs des Sciences et des Arts, Paris, and thoroughly reviewed back then ([Zbl 0102.25503](#)).

Chapter 9 comes with the title "Sesquilinear Forms and Quadratic Forms" and systematically develops the conceptual framework of this central topic in modern (multi-)linear algebra, together with a few applications to linear analytic geometry at the end.

Almost fifty years ago, this systemating, consequently structural approach to quadratic, symplectic or hermitian forms and their transformation groups was entirely new and pioneering, with a strong influence on the subsequent textbook literature thereafter. In our days, most of the material treated in this chapter is common standard in most advanced linear algebra courses, which just bespeaks the particular significance that this classic once had (and still has). Also, this Chapter 9 of Bourbaki's "Algebra" still breathes Jean Dieudonné's glaring mathematical spirit, his masterly style of exposition, and his unchallenged expertise in history of mathematics in a conspicuous manner.

As for the precise contents, Chapter 9 consists of 10 sections, each of which is divided into several subsections. Section 1 treats the general theory of sesquilinear forms, whereas Section 2 is devoted to the concept of discriminant of a sesquilinear form. Section 3 turns to hermitian forms and their associated quadratic forms, including the notion of so-called ε -hermitian forms. Section 4 deals then with totally isotropic subspaces and E. Witt's fundamental decomposition theorem. Alternating bilinear forms, Pfaffians, and the symplectic group are the main objects of study in Chapter 5, followed by a thorough discussion of particular properties of hermitian forms, unitary groups, orthogonal groups, and hermitian geometry. Section 7 treats hermitian forms over ordered fields (e.g., symmetric forms over \mathbb{R}), hermitian metrics, Sylvester's law of inertia, and the allied reduction theory.

Section 8 discusses types of quadratic forms over a commutative ring, and Clifford algebras are the subject of Section 9. The concluding Section 10 gives some geometric interpretations and applications of the theory of quadratic forms (e.g., affine similarity transformations of the plane, plane trigonometry, angles in the plane, and angular sectors). This chapter ends with a historical note of 12 pages, presumably composed by J. Dieudonné in his typical, very cultured style. As usual for the Bourbaki volumes, each section is enhanced by a large number of complementing, difficult, and further-leading exercises, most of which provide instructive examples, counter-examples, or additional theorems.

Like most of the Bourbaki volumes, also this Chapter 9 of the algebra series must be seen as a historical milestone, and therefore it is utmost gratifying, to have a reprinting of it available for further generations of mathematicians.

Reviewer: [Werner Kleinert \(Berlin\)](#)

MSC:

- 13-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
- 11-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
- 15-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to linear algebra
- 11E04 Quadratic forms over general fields
- 13A50 Actions of groups on commutative rings; invariant theory
- 15A63 Quadratic and bilinear forms, inner products

Cited in **2** Reviews
Cited in **45** Documents

Keywords:

textbooks; quadratic forms over general fields; bilinear algebra; hermitian forms; symplectic forms; linear groups; invariant theory; plane geometry