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On quasi-strongly regular graphs. (English) Zbl 1108.05097

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Summary: We study the quasi-strongly regular graphs, which are a combinatorial generalization of the strongly regular and the distance regular graphs. Our main focus is on quasi-strongly regular graphs of grade 2. We prove a "spectral gap"-type result for them which generalizes Seidel's well-known formula for the eigenvalues of a strongly regular graph [see *J. J. Seidel*, Linear Algebra Appl. 1, 281–298 (1968; Zbl 0159.25403)]. We also obtain a number of necessary conditions for the feasibility of parameter sets and some structural results. We propose the heuristic principle that the quasi-strongly regular graphs can be viewed as a "lower-order approximation" to the distance regular graphs. This idea is illustrated by extending a known result from the distance-regular case to the quasi-strongly regular case. Along these lines, we propose a number of conjectures and open problems. Finally, we list all the proper connected quasi-strongly graphs of grade 2 with up to 12 vertices.

MSC:

05E30 Association schemes, strongly regular graphs
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
15A42 Inequalities involving eigenvalues and eigenvectors

Keywords:

distance regular graph; adjacency eigenvalues; spectral gap; feasibility conditions

Full Text: DOI

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