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On Hamiltonian perturbations of hyperbolic systems of conservation laws. I: Quasi-triviality of bi-Hamiltonian perturbations. (English) Zbl 1108.35112

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Summary: We study the general structure of formal perturbative solutions to the Hamiltonian perturbations of spatially one-dimensional systems of hyperbolic PDEs $v_t + [\phi(v)]_x = 0$. Under certain genericity assumptions it is proved that any bi-Hamiltonian perturbation can be eliminated in all orders of the perturbative expansion by a change of coordinates on the infinite jet space depending rationally on the derivatives. The main tool is in constructing the so-called quasi-Miura transformation of jet coordinates, eliminating an arbitrary deformation of a semisimple bi-Hamiltonian structure of hydrodynamic type (the quasi-triviality theorem). We also describe, following *S.-Q. Liu* and *Y. Zhang* [*J. Geom. Phys.* 54, 427–453 (2005; Zbl 1079.37058)], the invariants of such bi-Hamiltonian structures with respect to the group of Miura-type transformations depending polynomially on the derivatives

MSC:

35L65 Hyperbolic conservation laws

37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)

Cited in **2** Reviews
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