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NP-containment for the coherence test of assessments of conditional probability: a fuzzy logical approach. (English) Zbl 1110.03012

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Summary: In this paper we investigate the problem of testing the coherence of an assessment of conditional probability following a purely logical setting. In particular we prove that the coherence of an assessment of conditional probability χ can be characterized by means of the logical consistency of a suitable theory T_χ defined on the modal-fuzzy logic $FP_k(R\text{Ł}_\Delta)$ built up over the many-valued logic $R\text{Ł}_\Delta$. Such modal-fuzzy logic was previously introduced by the author [Lect. Notes Comput. Sci. 3571, 714–725 (2005; Zbl 1109.03018)] in order to treat conditional probability by means of a list of simple probabilities following the well-known (smart) ideas exposed by *J. Y. Halpern* ["Lexicographic probability, conditional probability, and nonstandard probability", in: Proceedings of the eighth conference on theoretical aspects of rationality and knowledge, 17–30 (2001)] and by *G. Coletti* and *R. Scozzafava* [Probabilistic logic in a coherent setting. Dordrecht: Kluwer (2003; Zbl 1040.03017)]. Roughly speaking, such logic is obtained by adding to the language of $R\text{Ł}_\Delta$ a list of k modalities for "probably" and axioms reflecting the properties of simple probability measures. Moreover we prove that the satisfiability problem for modal formulas of $FP_k(R\text{Ł}_\Delta)$ is NP-complete. Finally, as main result of this paper, we prove that the problem of establishing the coherence of rational assessments of conditional probability is NP-complete.

MSC:

03B48 Probability and inductive logic
03B52 Fuzzy logic; logic of vagueness
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Keywords:

conditional probability; coherence; computational complexity; rational Łukasiewicz logic; Baaz connective; modal-fuzzy logic; satisfiability problem; NP-complete

Full Text: DOI

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