Dixon, John D.; **Rahnamai Barghi, A.**
Irreducible characters which are zero on only one conjugacy class. (English) Zbl 1112.20007
Proc. Am. Math. Soc. 135, No. 1, 41-45 (2007).

Let G be a finite group and let χ be an irreducible complex character of G of degree greater than 1. A theorem of Burnside shows that χ vanishes on at least one conjugacy class of G. As its title suggests, this paper is concerned with investigating what can be said when χ vanishes on exactly one conjugacy class. The available examples suggest that the phenomenon is related to certain doubly transitive groups. For example, when q is a power of a prime p, the Steinberg character of the group $\mathrm{PGL}(2,q)$ is an irreducible character of degree q, derived from a doubly transitive action on $q+1$ points, which vanishes only on elements of order p. Since all elements of order p are conjugate in $\mathrm{PGL}(2,q)$, we have a group with the required property, and this group is simple when q is a power of 2. There are one or two other simple groups with the unique conjugacy class property, but we do not know of any infinite families except that described above.

The authors' main theorem is that if G is a finite solvable group with a character of the type described above, it has a homomorphic image which is a doubly transitive group. In this case, $\chi(1)+1$ is a power of a prime. This result is deduced from a more general hypothesis concerning the existence of an Abelian chief factor in G, which automatically holds if G is solvable. It seems that more might be said, as the role of the kernel of χ is not fully exploited. For if $\chi(g)=0$ and h is any element of $\ker\chi$, then $\chi(gh)=0$ also, and hence g and gh are conjugate in G. This is a condition which may tell us something about the structure of $\ker\chi$ or the action of G on this normal subgroup. For example, if g is an involution, the kernel must be Abelian.

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MSC:

20C15 Ordinary representations and characters

Keywords:

characters; solvable groups; doubly transitive groups

Software:

GAP

Full Text: DOI

References:

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