

**Marcovecchio, Raffaele**

**Linear independence of linear forms in polylogarithms.** (English) Zbl 1114.11063

Ann. Sc. Norm. Super. Pisa, Cl. Sci. (5) 5, No. 1, 1-11 (2006).

For a complex number  $x$  with  $|x| < 1$  and a positive integer  $s$ , the  $s$ -th polylogarithms of  $x$  are  $Li_s(x) = \sum_{k=1}^{\infty} x^k/k^s$ . The purpose of this paper is to prove that for any non-zero algebraic number  $\alpha$  with  $|\alpha| < 1$  in the sequence of  $1, Li_1(\alpha), Li_2(\alpha), \dots$  infinitely many terms are linearly independent over  $\mathbb{Q}(\alpha)$ . This result extends a previous one for rational  $\alpha$  by *T. Rivoal* [J. Théor. Nombres Bordx. 15, No. 2, 551–559 (2003; Zbl 1079.11038)]. The main tool is a method introduced by *S. Fischler* and *T. Rivoal* [J. Math. Pures Appl. (9) 82, No. 10, 1369–1394 (2003; Zbl 1064.11053)], which shows that the vector of the coefficients of the polylogarithms in the relevant series is the unique non-zero solution (up to a multiplicative constant) of a suitable Padé approximation problem.

Reviewer: [Takao Komatsu \(Hirosaki\)](#)

**MSC:**

- [11J72](#) Irrationality; linear independence over a field
- [11J17](#) Approximation by numbers from a fixed field
- [11J91](#) Transcendence theory of other special functions
- [11G55](#) Polylogarithms and relations with  $K$ -theory

Cited in **1** Review  
Cited in **9** Documents

**Full Text:** [EuDML](#)

**References:**

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