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Intersection patterns of essential surfaces in 3-manifolds. (English) Zbl 1115.57301
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It is a key conjecture in the theory of 3-manifolds that every closed, orientable, irreducible 3-manifold M with infinite fundamental group contains an immersed, closed, orientable, incompressible surface, or equivalently, that its fundamental group contains the fundamental group of a closed, orientable surface. Such an immersed surface is called essential in the present paper. For example, if M is virtually Haken (finitely covered by a Haken manifold) then it contains an essential surface, but the converse is not known at present (that is, if every essential surface lifts to an embedded surface in some finite covering of M). Thus “separation properties of essential surfaces in finite coverings are key properties to study”. An essential surface is said to have the finite plane intersection property if there exists a k such that, after a homotopy, every collection of k lifts of the surface in the universal cover of M contains a pair that does not intersect.

The main result of the present paper is that every essential geometrically finite surface in a finite volume hyperbolic 3-manifold satisfies the finite plane intersection property (if the surface is not geometrically finite, it is a virtual fiber). It is also proved that, for an essential surface in a closed hyperbolic 3-manifold M , the covering of M corresponding to the fundamental group of the surface has a finite cover in which every lift of the surface is embedded.

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