

**Gillet, Henri; Grayson, Daniel R.**

**Volumes of symmetric spaces via lattice points.** (English) Zbl 1116.11028  
*Doc. Math.* 11, 425-447 (2006).

It is shown how to use elementary methods to prove that the volume of  $SL_k(\mathbb{R})/SL_k(\mathbb{Z})$  is  $\zeta(2)\zeta(3)\dots\zeta(k)/k$ . Using a version of reduction theory, we can compute the volumes of certain unbounded regions in Euclidean space by counting lattice points and using the machinery of Dirichlet series to get an estimate of the growth rate of the number of lattice points appearing in the region as the lattice spacing decreases. Some of the techniques used here were known to Siegel (1935–1937) in his investigation of representability of integers by quadratic forms. A closely related result that the Tamagawa number of  $SL(k, \mathbb{Q})$  is 1 is also proved in a simpler and more arithmetic manner than the proof of Weil (1982)

Reviewer: [Ranjeet Sehmi \(Chandigarh\)](#)

**MSC:**

[11F12](#) Automorphic forms, one variable  
[11H06](#) Lattices and convex bodies (number-theoretic aspects)  
[11M45](#) Tauberian theorems

Cited in **1** Document

**Keywords:**

volume; lattice points; Tamagawa number; Dirichlet series

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