Báez-Duarte, Luis

A general strong Nyman-Beurling criterion for the Riemann hypothesis. (English)

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The author considers the Müntz operator

$$g(x)=Pf(x):=\sum_{n\geq 1} f(nx)-\frac{1}{x}\int_0^\infty f(t)\,dt$$

for suitable f. For certain f with both $f,g\in L_2(0,\infty)$ it is true that the famous Riemann Hypothesis (all complex zeros of the Riemann zeta-function $\zeta(s)$ have real parts equal to $1/2$) is equivalent to the fact that f is in the L_2 closure of the vector space generated by the dilatations $g(kx)$ when $k\in\mathbb{N}$. In this paper the author obtains, by methods from functional analysis, additional equivalent statements of the Riemann Hypothesis. An essential rôle is played by the classical representation

$$\zeta(s)=s\int_0^\infty \frac{[x]-x}{x^{s+1}}\,dx \qquad (0<\sigma<1,\ s=\sigma+it),$$

where $[x]$ is the integer part of x. This is a special case of the Müntz formula, which says that under suitable conditions

$$\zeta(s)\widehat{f}(s)=\widehat{Pf}(s) \qquad (0<\sigma<1,\ s=\sigma+it),$$

where $\widehat{f}(s)=\int_0^\infty x^{s-1}f(x)\,dx$ is the usual Mellin tranform of f.

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MSC:

11M06 $\zeta(s)$ and $L(s,\chi)$
11M26 Nonreal zeros of $\zeta(s)$ and $L(s,\chi)$; Riemann and other hypotheses

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