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On 2-absorbing ideals of commutative rings. (English) Zbl 1120.13004

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Summary: Suppose that R is a commutative ring with $1 \neq 0$. We introduce the concept of 2-absorbing ideal which is a generalisation of prime ideal. A nonzero proper ideal I of R is called a 2-absorbing ideal of R if whenever $a, b, c \in R$ and $abc \in I$, then $ab \in I$ or $ac \in I$ or $bc \in I$. It is shown that a nonzero proper ideal I of R is a 2-absorbing ideal if and only if whenever $I_1I_2I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of R, then $I_1I_2 \subseteq I$ or $I_2I_3 \subseteq I$ or $I_1I_3 \subseteq I$. It is shown that if I is a 2-absorbing ideal of R, then either $\mathrm{Rad}(I)$ is a prime ideal of R or $\mathrm{Rad}(I) = P_1 \cap P_2$ where P_1, P_2 are the only distinct prime ideals of R that are minimal over I. Rings with the property that every nonzero proper ideal is a 2-absorbing ideal are characterised. All 2-absorbing ideals of valuation domains and Prüfer domains are completely described. It is shown that a noetherian domain R is a Dedekind domain if and only if a 2-absorbing ideal of R is either a maximal ideal of R or M^2 for some maximal ideal M of R or M_1M_2 where M_1, M_2 are some maximal ideals of R. If R_M is noetherian for each maximal ideal M of R, then it is shown that an integral domain R is an almost Dedekind domain if and only if a 2-absorbing ideal of R is either a maximal ideal of R or M^2 for some maximal ideal M of R or M_1M_2 where M_1, M_2 are some maximal ideals of R.

MSC:

13A15 Ideals and multiplicative ideal theory in commutative rings

Full Text: DOI

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