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Estimates to the stability of functional equations. (English) Zbl 1121.39029
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The stability of Cauchy's functional equation is studied under some general circumstances. Given two binary operations, one in the domain and the other in the range. Is is extensively discussed what properties of these operations, the sets in the domain and range and of the upper bound of the "Cauchy difference" imply the stability behaviour. The estimates of the pointwise distance between the solution and the approximate solution of the equation are also obtained (however, they could not be optimal).

The following idea due to P. Volkmann is used. Given the Cauchy equation

$$g(x \diamond y) = g(x) * g(y) \tag{1}$$

and the stability problem

$$d\big(f(x \diamond y), f(x) * f(y)\big) \le \varepsilon(x, y) \tag{2}$$

(where d is some metric in the range). Putting $y = x$ to (1), (2) we obtain the stability problem in a single variable: (1) becomes

$$g(x \diamond x) = g(x) * g(x), \tag{3}$$

while (2) has the form

$$d\big(f(x \diamond x), f(x) * f(x)\big) \le \varepsilon(x, x). \tag{4}$$

Now, assuming (4), we find the solution g of (3) which is close (in some sense) to f, i.e.

$$d\big(f(x), g(x)\big) \le \Phi(x). \tag{5}$$

Then the suitable properties of the binary operations involved imply that, assuming (2), g is also the solution of (1) fulfilling (5).

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MSC:

39B82 Stability, separation, extension, and related topics for functional equations
39B52 Functional equations for functions with more general domains and/or ranges

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