Diagana, Toka

Algebraic sum of unbounded normal operators and the square root problem of Kato.
(English) Zbl 1121.47015

Rend. Semin. Mat. Univ. Padova 110, 269-275 (2003).

Taking A and B to be unbounded normal operators on a complex Hilbert space $\mathbb{H}$, we may, as a consequence of the spectral theorem, write $A = A_1 - iA_2$ and $B = B_1 - iB_2$, where the A_k and B_k are unbounded selfadjoint operators on $\mathbb{H}$. If, moreover, the operators A_k and B_k are non-negative, then, considering the sesquilinear functionals defined by $\phi(u, v) := \langle A_1 u, v\rangle - i\langle A_2 u, v\rangle$, $\psi(u, v) := \langle B_1 u, v\rangle - i\langle B_2 u, v\rangle$, and $\xi(u, v) := \phi(u, v) + \psi(u, v)$, we see that if ϕ and ψ are sectorial, that is, if there exist constants c_1 and $c_2$with $\operatorname{Im}\phi(u, u) \le c_1 \operatorname{Re}\phi(u, u)$ and $\operatorname{Im}\psi(u, u) \le c_1 \operatorname{Re}\psi(u, u)$ for all u in the appropriate domains, then ξ is sectorial as well.

Theorem 2.1 of the present paper demonstrates that, under the further assumptions that the intersection of the domains of A and B is dense in $\mathbb{H}$ and that the operator $\overline{A+B}$ is maximal, then this latter operator satisfies the square root problem of Kato; that is, the domains of $\overline{A+B}^{1/2}$ and $\overline{A+B}^{*1/2}$ both coincide with the intersection of the domains of $A^{1/2}$ and $B^{1/2}$. The density assumption on $\mathrm{Dom}(A) \cap \mathrm{Dom}(B)$ can be replaced with certain conditions on $\mathrm{Dom}(|A|^{1/2}) \cap \mathrm{Dom}(|B|^{1/2})$ that ensure (Theorem 2.2) the existence of an operator $A \oplus B$ (a "generalized" sum of A and B) satisfying the square root problem of Kato. (So $\mathrm{Dom}((A \oplus B)^{1/2}) = \mathrm{Dom}((A \oplus B)^{*1/2}) = \mathrm{Dom}(|A|^{1/2}) \cap \mathrm{Dom}(|B|^{1/2})$.)

The paper concludes with an example where the sum $A + B$ is a perturbed Schrödinger operator.

Reviewer: Timothy Feeman (Villanova)

MSC:

47B25 Linear symmetric and selfadjoint operators (unbounded)
35J15 Second-order elliptic equations
47D08 Schrödinger and Feynman-Kac semigroups

Full Text: EuDML arXiv

References:

[1] P. AUSHER - S. HOFMANN - A. MCINTOSH - P. TCHAMITCHIAN, The Kato Square Root Problem for Higher Order Elliptic Operators and Systems on Rn , J. Evol. Eq., 1, No. 4 (2001), pp. 361-385. Zbl1019.35029 MR1877264 · Zbl 1019.35029 · doi:10.1007/PL00001377

[2] A. BIVAR-WEINHOLTZ - M. LAPIDUS, Product Formula for Resolvents of Normal Operator and the Modified Feynman Integral, Proc. Amer. Math. Soc., 110, No. 2 (1990). Zbl0718.47023 MR1013964 · Zbl 0718.47023 · doi:10.2307/2048088

[3] H. BRÉZIS - T. KATO, Remarks on the Schrödinger Operator with Singular Complex Potentials, J. Math. Pures Appl., 58 (1979), pp. 137-151. Zbl0408.35025 MR539217 · Zbl 0408.35025

[4] T. DIAGANA, Sommes d'opérateurs et conjecture de Kato-McIntosh, C. R. Acad. Sci. Paris, t. 330, Série I (2000), pp. 461-464. Zbl0951.47001 MR1756959 · Zbl 0951.47001 · doi:10.1016/S0764-4442(00)00200-7

[5] T. DIAGANA, Schrödinger operators with a singular potential, Int. J. MathMath. Sci., 29, No. 6 (2002), pp. 371-373. Zbl0997.35010 MR1897865 · Zbl 0997.35010 · doi:10.1155/S0161171202007330

[6] T. DIAGANA, Quelques remarques sur l'opérateur de Schödinger avec un potentiel complexe singulier particulier, Bull. Belgian. Math. Soc., 9 (2002), pp. 293-298. Zbl1040.35017 MR2017083 · Zbl 1040.35017

[7] T. DIAGANA, An application to Kato's square root problem, Int. J. MathMath. Sci. Vol., 9, No. 3 (2002), pp. 179-181. Zbl1001.47021 MR1888351 · Zbl 1001.47021 · doi:10.1155/S0161171202007706

[8] T. DIAGANA, A Generalization related to Schrödinger operators with a singular potential, Int. J. Math-Math. Sci., 29, No. 10 (2002), pp. 609-611. Zbl1085.35051 MR1900505 · Zbl 1085.35051 · doi:10.1155/S0161171202007974

[9] T. KATO, Perturbation theory for linear operators, New york (1966). Zbl0148.12601 MR203473 · Zbl 0148.12601

[10] J. L. LIONS, Espace intermédiaires entre espaces Hilbertiens et applications, Bull. Math. Soc. Sci. Math. Phys. R. P. Roumanie (N.S), 2 (50) (1958), pp. 419-432. Zbl0097.09501 MR151829 · Zbl 0097.09501

[11] J. L. LIONS, Espaces d'interpolation et domaines de puissances fractionnaires d'opérateurs, J. Math. Soc. Japan., 14 (2) (1962). Zbl0108.11202 · Zbl 0108.11202 · doi:10.2969/jmsj/01420233

[12] A. PAZY, Semigroups of linear operators and application to partial differential equations, Springer-Verlag, New York (1983). Zbl0516.47023 MR710486 · Zbl 0516.47023

[13] W. RUDIN, Functional analysis, Tata McGraw-Hill, New Delhi (1974). Zbl0253.46001 MR365062 · Zbl 0253.46001