A submersion $\pi: M \to B$ is called Riemannian if $M$ and $B$ are Riemannian manifolds and the tangential $\pi^*$ preserves lengths of horizontal vectors. The case of totally geodesic fibers $\pi^{-1}(p)$, $p \in B$, has been dealt with in R. H. Escobales jun. [J. Differ. Geom. 10, 253–276 (1975; Zbl 0301.53024)]. The present papers discusses minimal fibers. In more detail:

1. If $M$ has positive curvature, then the horizontal distribution is non-totally geodesic.
2. If $M$ has negative curvature, then the fibers have negative scalar curvature.

Both results are deduced from more quantitative relations, expressing sums of sectional curvatures for mixed vertical-horizontal, resp., pure vertical tangent planes by certain other invariants, in particular the scalar curvature of the fibers. Also the case of a manifold $M$ with non-positive curvature and fibers with zero scalar curvature is considered, both properties together lead to flat spaces $M, B$, and local triviality of $\pi$.

In second part of the paper, the setting of a Riemannian submersion $\pi: M \to B$ with minimal fibers is applied to centroaffine hypersurface theory, i.e., to a hypersurface immersion $\Phi: M \to \mathbb{R}^{n+1}$ with $\Phi$ (or $-\Phi$) as a transversal normalization, where $M$ obtains the centroaffinely-induced metric, assumed to be positive definite. In the realm of centroaffine hypersurfaces the (equiaffine) hyperspheres can be characterized by the vanishing of the Chebyshev vector field. A typical result of this part is:

3. If $\Phi$ is an elliptic proper centroaffine hypersphere then the submersion has non-totally geodesic horizontal distribution. This time, an inequality between certain invariants, including the squared norm of the Chebyshev vector field is basic for this type of result.

There are more applications and corollaries, e.g., on conditions for the non-realizability of $M$ as a proper elliptic or improper hypersphere.

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