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$K3$ surfaces with Picard number one and infinitely many rational points. (English)

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The author constructs infinitely many $K3$-surfaces $X/\mathbb{Q}$ such that $X(\mathbb{Q})$ is infinite and the geometric Picard number of X equals 1. A sketch of the proof is as follows. The author starts with a family of degree 4 surfaces X in $\mathbb{P}^2$, such that all members of this family reduce to the same surface modulo 6.

Let X_p denote the reduction of X modulo p. Using the Lefschetz trace formula the author shows that the geometric Picard number of X_2 and of X_3 equal two. This implies that the geometric Picard number of X is at most 2. He shows that X_2 contains a line and X_3 contains a conic. This implies that the discriminant of the Néron-Severi lattice of X_2 and of X_3 differ by a non-square. He then observes that if X would have geometric Picard number 2 then both discriminants would differ by a square, hence X has geometric Picard number 2.

He considers a subfamily of X such that each member has a hyperplane section with two nodes. The normalization E of such a curve is an elliptic curve. The author finds two rational points O, T on E, taking O as the identity he checks whether $2520T = O$, since this is not the case it follows from a Theorem of Mazur that T has infinite order, hence X contains infinitely many rational points.

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