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**PDAE models of integrated circuits and index analysis.** (English) Zbl 1123.78009

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In section 2 the authors consider an RLC network with one semiconductor modeled by the stationary drift diffusion equations. The circuit has $n+1$ nodes and contains semiconductors, resistors, inductors, capacitors and independent voltage and current sources denoted $\{S, R, L, C, V, I_i\}$, respectively. By $k_E \in N$ with $E$ from the index set above the number of the elements $E$ in the circuit is denoted. Also $k_{I_c}$ voltage controlled current sources $I_c$ connected parallel to capacitors are considered. These appear in, for instance, diode equivalent circuits for $pn$-functions.

The topology of the network is defined through the incidence matrices $A_E \in \mathbb{R}^{n\times k_E}$. The inputs of the system are the functions $i_s(\cdot) \in \mathbb{R}^{k_I}$ and $v_s(\cdot) \in \mathbb{R}^{K_v}$ describing the behavior of the independent sources $I_i$ and $V$.

In the modified nodal analysis (MNA), Kirchhoff's laws and specific relations describing the network elements are combined in a differential algebraic equation (DAE). The unknowns are reduced to a vector $x(t) = (e(t), i_L(t), i_V(t) \in \mathbb{R}^{n+k_I+k_V}$ containing the node potentials, the currents through the inductors and the currents through the voltage sources

$$0 = A_C q_C(A_C^T e, t)^j + A_R g(A_R^T e, t) + A_L i_L(t) + A_V i_V + A_{I_c} i_I(A_{I_c}^T e, t) + A_{I_i} i_S + A_S j_s,$$
$$0 = \Phi(i_L(t), t)^j - A_L^T E,$$
$$0 = A_V^T e - v_s(t).$$

The functions $q_C$ and $\Phi$ are the electric charges and electromagnetic fluxes, respectively, and $g$ and $\Phi$ describe the voltage dependence of resistances and controlled current sources, respectively.

It is assumed that the resistors, inductors and capacitors are locally passive, i.e. the matrices

$$G(t) = \frac{\partial g(w,t)}{\partial w}, \quad L(t) = \frac{\partial \Phi(w,t)}{\partial w}, \quad C(t) = \frac{\partial q_C(w,t)}{\partial w}$$

are weakly (not necessarily symmetric) positive definite.

The authors motivate the choice of the MNA equations to model the dynamics of the electric circuit by the fact that the MNA equations have a relatively small number of unknowns and can be set up automatically – two important features for the development of digital memory circuits having up to $10^7$ network components. Furthermore, the MNA equations lead to differential algebraic equations at most index 2 if all capacitances and inductances are passive and controlled sources satisfy weak topological assumptions concerning their controlling voltages and currents. The higher index variables depend only linearly on the other network variables. It implies that the weak instability known for higher index differential algebraic equations to be harmless in case of network differential algebraic equations formulated by MNA.

The drift diffusion equations and the boundary conditions are also presented in the section 2. The coupled system and the generalisation of the tractability index are presented in sections 3 and 4. The greater part of the index proof consists of an existence proof for the linearized drift diffusion equations contained in section 5.

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