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Relationships among some locally conservative discretization methods which handle discontinuous coefficients. (English) Zbl 1124.76030

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Summary: This paper presents the relationships between some numerical methods suitable for a heterogeneous elliptic equation with application to reservoir simulation. The methods discussed are the classical mixed finite element method (MFEM), the control-volume mixed finite element method (CVMFEM), the support operators method (SOM), the enhanced cell-centered finite difference method (ECCFDM), and the multi-point flux-approximation (MPFA) control-volume method. These methods are all locally mass conservative, and handle general irregular grids with anisotropic and heterogeneous discontinuous permeability. In addition to this, the methods have a common weak continuity in the pressure across the edges, which in some cases corresponds to Lagrange multipliers. It seems that this last property is an essential common quality for these methods.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
76S05 Flows in porous media; filtration; seepage
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N06 Finite difference methods for boundary value problems involving PDEs

Keywords:

relationships, mixed finite element method (MFEM); expanded mixed finite element method (EMFEM); enhanced cell-centered finite difference method (ECCFDM); control-volume mixed finite element method (CVMFEM); support operator method (SOM); multi-point flux-approximation (MPFA) control-volume method

Full Text: DOI

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