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Quantum field theory. I: Basics in mathematics and physics. A bridge between mathematicians and physicists. (English) Zbl 1124.81002

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This is the first volume of a six volume book on quantum field theory. Other volumes are planned to be; Volume II: Quantum electromagnetism, Volume III: Gauge theories, Volume IV: Quantum mathematics, Volume V: The physics of the standard model, Volume VI: Quantum gravity and string theory. This first volume deals with basic mathematical tools and physics of quantum field theory and aims to bridge mathematicians and physicists.

In the introduction, the author quotes: "I don't understand how anyone can teach mathematics without having a battery of problems that the students are going to be inspired to want to solve" (Weinberg, 1986). This book is written in accord with this opinion. The author says "The most important notion of modern physics is the Feynman functional integral as a partition function for the states of many-particle systems" (p. 15). Three magic formulas lie at the heart of quantum field theory: 1. the Dyson formula 2, the Feynman propagator formula, and 3. the Gell-Mann-Low formula.

In Chapter 7, finite-dimensional versions of these formulas are rigorously proved. This is one of the highlights of this book. Rigorous proofs of these formulas in its original form are still a challenging problem for mathematicians. Part III of this book is devoted to heuristic "proofs" of these formulas due to physicists. But the object of this book is not limited to derive these formulas. The main object is to explain mathematics for students or researchers in physics how to use them, and to show students and reserachers in mathematics how to use mathematics in physics. Translations of different languages used by mathematicians and physicists for the same mathematical objects are also presented.

The book is divided into three parts: Part I. Introduction, Part II. Basic techniques in mathematics, Part III. Heuristic magic formulas of quantum ield theory.

Part I deals with the history of physics after Planck and Einstein (Chap. 1) quoting many testimonies of several scholars. Then phenomenology of the standard model for elementary particles and the trouble with scale change (challenge of different scale in nature) are exposed in Chap. 2 and 3. (Charge conjugation in p. 173. $(C\psi)(x,t) := \psi(x,t)$ is a misprint $-\psi(x,t)$). They introduce and give overviews of the response function, the Feynman propagator, causality, the role of symmetry in physics and renormalization together with Fourier and Laplace transformation, and modern differential geometry. Part I is concluded to glance at conformal field theory.

Part II is the core of this book. It consists of 12 chapters. The first 3 chapters aim to prepare elements of mathematical tools which are continued by analytic functions (Chap. 4), topology and characteristic classes (Chap. 5), and number theory (Chap. 6).

Topics and explanations of Chap. 4 and 5 seem rather usual, while Chap. 6 seems interesting even for pure mathematicians. It deals with Riemann's zeta functions as the tool to study partition problems, which is important both in number theory and statistical physics of many particle systems. Then the technique of zeta-regularization is explained and it is remarked that the regularized value $\zeta(1) = -\frac{1}{12}$ is observed in the experiments of the Casimir effect (6.6). As already mentioned, Chapter 7 contains rigorous proofs of central formulas in the finite dimensional case. These proofs present algebraic (formal) parts of derivations of these formulas. But true difficulties in derivations of these formulas are topological problems; that is, passages from finite dimensions to infinite dimensions. In Part III of this book, physicist's methods of such passeges are explained. But their mathematical justifications are still challenging problems for mathematicians. This chapter also contains some infinite-dimensional problems, such as the zeta function description of Gaussian path integrals (7.23), and explanations of Lagrange multipliers (7.28).

The next two chapters deal with rigourous finite-dimensional perturbation theory [cf. *T. Kato*, A short introduction to perturbation theory for linear operators of Springer (1982; Zbl 0836.47009)] together with explanations on the Baker-Campbell-Hausdorff formula and regularization procedures used in the Weierstrass product formula and so on (Chap. 8). Moreover, the calculus for Grassmann variables focussing its application to physics of fermions is considered (Chap. 9). Chap. 8 also aims to give the first course of renormalization for the readers.

The rest of Part II is devoted to the explanations of infinite-dimensional mathematics, which are indispensable in the study of quantum field theory. In Chap. 10, the infinite dimensional Hilbert space is applied to the Dirichlet problem. Starting from expositions of formal Dirac calculus, the notion of distribution is introduced to give a rigourous mathematical justification of Dirac calculus in Chap. 11. Chap. 12, the last Chapter of Part II begins to expose the discrete Dirac calculus. Then, introducing a Gelfand triple $\mathcal{S}(\mathbb{R}) \subset L_2(\mathbb{R}) \subset \mathcal{S}'(\mathbb{R})$, a rigorous meaning of formal Dirac calculus is given (12.2). This Section also contains expositions on spectral theorems.

Part III summarizes important heuristic formulas used in quantum field theory. They are derived as the formal continuum limits of rigorous mathematics (p. 740). The author summarizes the advantages and disadvantages of two approaches to write n-point functions as follows: Correlation functions depend only on the classical action, but the functional integral is not a well-defined mathematical object. On the other hand, the functional integral does not appear in the Green's functions. They are operators in Hilbert space. But the operators $\varphi(x)$ are highly singular objects and the commutation (or anticommutation) relations are not known a priori.

In the 1950s, physicists developed magic formuals in order to pass from the S-matrix to the correlation functions. Full quantum field can be computed as a functional derivative of the S-matrix formulated in the language of tempered distributions (Bogoliubov's formula, (15.35)). These magic formulas are shown in Chap. 14 (the response approach) and 15 (the operator approach).

The response approach (Chap. 14) starts from the classical principle of critical action, and derives the classical field equation $D\varphi = -\kappa\mathfrak{L}_{\text{int}}(\varphi) - J$, J is an arbitrary external source. Setting $\kappa = 0$, the response equation $D\varphi = -J$ is obtained. This Chapter is concluded by a sketch how to apply the response approach at quantum electrodynamics.

Chap. 15 deals with the operator approach taking the φ^4-model as an example. After computing the free 2-point Green's function by using Fourier expansion and regularization parameters, the magic Dyson formula on the finite time segment $[-\frac{T}{2}, \frac{T}{2}]$ and the full n-point Green's function are derived. This Chapter also contains an overview on renormalization, including remarks on its Hopf algebra symmetry and the Galois group of renormalization due to Kreimer, Connes and Marcolli. The relation between Dyson's no-go argument and Ritt's theorem, which asserts any formal power series can be realized as an asymptotic series of some holomorphic function (Th. 15.9). [*J. F. Ritt*, "On the derivaitve of a function at a point, Ann. Math. (2) 18, 18–23 (1916; JFM 46.0471.02)] is also remarked.

Chap. 16 deals with gauge theory focussing its cohomological nature. Languages of modern differential geometry and cohomology are first explained in this Chapter, together with the dictionary of Wu-Yang. In Chap. 17, the last Chapter of this book, a panorama of literatures is presented. It is a large list, but for the further search of literature, the author recommends the site `http://www.mis.mpg.de/`. This chapter also contains a glance on the monster group and vertex algebras. A look at the history of string theory is also given.

In the Appendix, lists of notations, physical units, and brief expositions on dimensional analysis are presented. In the Epilogue, several scholar opinions on the relations of mathematics to physics from Bacon, Pascal, through von Neumann, Weinberg, are collected.

Reviewer: Akira Asada (Takarazuka)

MSC:

81-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to quantum theory
81Txx Quantum field theory; related classical field theories
81Q05 Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
81Q30 Feynman integrals and graphs; applications of algebraic topology and algebraic geometry
81S40 Path integrals in quantum mechanics
81U20 S-matrix theory, etc. in quantum theory

Keywords:

Dyson formula for the S-matrix; Feynman propagator formula; Gell-Mann-Low formula; φ^4-model; cor-

relation function; Green's function

Full Text: DOI