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Perverse sheaves and Milnor fibers over singular varieties. (English) ｜Zbl 1131.32016｜

Brasselet, Jean-Paul (ed.) et al., Singularities in geometry and topology 2004. Proceedings of the 3rd Franco-Japanese colloquium on singularities, Hokkaido, Japan, September 13–18, 2004. Tokyo: Mathematical Society of Japan (ISBN 978-4-931469-39-6/hbk). Advanced Studies in Pure Mathematics 46, 211-222 (2007).

The aim of this note, says the author in the introduction, is to introduce some recent applications of perverse sheaves to the study of complex hypersurface singularities. Thus it is largely a survey article, but the proof of one result is given here for the first time.

The author first briefly recalls some general facts about Milnor fibres, in particular the vanishing theorem of Kato and Matsumoto for the reduced cohomology $\widetilde{H}^j(F_0, \mathbb{C})$ of the Milnor fibre F_0, outside the range $n - s \leq j \leq n$ where s is the dimension of the stratified singular locus. This theorem is later deduced quickly from general theorems about $\mathcal{D}$-modules and perverse sheaves. In between, he states and proves the theorem of the author and *P. Nang* [Math. Z. 249, No. 3, 493–511 (2005); addendum ibid. 250, No. 3, 729 (2005; Zbl 1066.14005); `math.AG/0410383`], generalising earlier results of Nang and the author and of Dimca. This bounds the number of Jordan blocks for a given eigenvalue of monodromy in terms of Betti numbers of complex links. Again the proof uses perverse sheaves and is quite short.

The ideas and main applications of $\mathcal{D}$-modules and perverse sheaves in the context of singularity theory are briskly but clearly outlined here, and the paper is written in an approachable way, assuming the minimum of technical knowledge.

For the entire collection see [Zbl 1111.58001].

Reviewer: G. K. Sankaran (Bath)

MSC:

32S40	Monodromy; relations with differential equations and D-modules (complex-analytic aspects)
14B05	Singularities in algebraic geometry
32C38	Sheaves of differential operators and their modules, D-modules
35A27	Microlocal methods and methods of sheaf theory and homological algebra applied to PDEs
32S55	Milnor fibration; relations with knot theory

Cited in **3** Documents

Keywords:

$\mathcal{D}$-module; perverse sheaf; Milnor fibre; monodromy