Kilbas, A. A.; **Titioura, A. A.**

Nonlinear differential equations with Marchaud-Hadamard-type fractional derivative in the weighted space of summable functions. (English) Zbl 1132.26314

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Summary: The paper is devoted to the study of a Cauchy-type problem for the nonlinear differential equation of fractional order $0<\alpha<1$,

$$(D^{\alpha}_{0+,\mu}y)(x)=f(x,y(x)),$$
$$(x^{\mu}\mathcal{J}^{1-\alpha}_{0+,\mu}y)(0+)=b,\quad b\in\mathbb{R},$$

containing the Marchaud-Hadamard-type fractional derivative $(D^{\alpha}_{0+,\mu}y)(x)$, on the half-axis $\mathbb{R}^{+}=(0,+\infty)$ in the space $X^{p;\alpha}_{c,0}(\mathbb{R}_{+})$ defined for $\alpha>0$ by

$$X^{p;\alpha}_{c,0}(\mathbb{R}_{+})=\{y\in X^{p}_{c}(\mathbb{R}_{+}):D^{\alpha}_{0+,\mu}y\in X^{p}_{c,0}(\mathbb{R}_{+})\}.$$

Here $X^{p}_{c,0}(\mathbb{R}_{+})$ is the subspace of $X^{p}_{c}(\mathbb{R}_{+})$ of functions g with compact support on infinity: $g(x)\equiv 0$ for large enough $x>R$. The equivalence of this problem and a nonlinear Volterra integral equation is established. The existence and uniqueness of the solution $y(x)$ of the above Cauchy-type problem is proved by using the Banach fixed point theorem. The solution in closed form of the above problem for the linear differential equation with $\{f(x,y(x))=\lambda y(x)+f(x)\}$ is constructed. The corresponding assertions for the differential equations with the Marchaud-Hadamard fractional derivative $(D^{\alpha}_{0+}y)(x)$ are presented. Examples are given.

MSC:

26A33 Fractional derivatives and integrals
34K30 Functional-differential equations in abstract spaces
34A12 Initial value problems, existence, uniqueness, continuous dependence and continuation of solutions to ordinary differential equations
45D05 Volterra integral equations
47N20 Applications of operator theory to differential and integral equations

Full Text: DOI