Levit, B.; Stepanova, N.
Efficient estimation of multivariate analytic functions in cube-like domains. (English)

Summary: Let $Q$ be an affine image of the unit cube in $\mathbb{R}^d$. An unknown function $f$ defined on $Q$ is observed in the continuous heteroscedastic regression model

$$dV(x) = f(x)dx + \varepsilon\sigma(x)dW(x), \quad x \in Q.$$ 

It is assumed that $f$ admits an analytic continuation into a certain vicinity $S_\gamma$ of $Q$ in the $d$-dimensional complex space $\mathbb{C}^d$. A special form of the noise variance $\sigma^2(x)$ ensures that the information about $f(x)$ provided by the data $V(\cdot)$ is (nearly) independent of $x \in \text{int} \, Q$. Projection-type estimates of $f(x)$ are shown to be asymptotically efficient for any given $x \in \text{int} \, Q$, as well as in all $L_p(Q)$-norms, $1 \leq p \leq \infty$, in both periodic and non-periodic cases.

MSC:
- 62G08 Nonparametric regression and quantile regression
- 62G20 Asymptotic properties of nonparametric inference
- 62H12 Estimation in multivariate analysis

Keywords: nonparametric estimation; heteroscedastic white noise model; multivariate analytic functions; projection-type estimates; Chebyshev-Fourier bases