Schmidli, Hanspeter

Stochastic control in insurance. (English) Zbl 1133.93002

Probability and its Applications. London: Springer (ISBN 978-1-84800-002-5/pbk). xv, 254 p. (2008).

In this book, the author discusses control theory and its application to insurance. In this regard, the author's view is that both life and non-life insurance provides fertile ground for the application of control techniques in a stochastic framework.

The book is arranged in four chapters together with an appendix consisting of six subsections.

The first chapter considers stochastic processes in discrete-time. In this framework, general results related to the dynamic programming principle and the optimal strategy are presented. The author applies the theory developed in this chapter to simple examples involving optimal dividend strategies in risk theory and the minimization of ruin probabilities.

In Chapter 2 stochastic processes in continuous-time are discussed. The author notes that, in many cases, continuous-time problems can be approximated by models in discrete time. However, the main results in this chapter is obtained via martingale methods with the connection with martingales being discussed in Section 2.1. The book highlights two main problems in the continuous-time paradigm, viz., problems related to the Hamilton-Jacobi-Bellman equation and the verification theorem. In addition, in this chapter, the author applies the developed theory to examples involving the minimization of ruin probabilities and optimization of dividends for both a diffusion approximation and a classical risk model.

The third chapter offers a consideration of problems from life insurance in a stochastic framework. In this regard, the author points out similarities between mathematical finance and life insurance and concludes that the latter is an appropriate field in which to apply stochastic control theory. Throughout this chapter, the classical Merton problem from mathematical finance is used as a backdrop for problems from life insurance. For the majority of these problems, the Hamilton-Jacobi-Bellman equation is not solvable so that the verification theorem has to be relied upon. In this context, the author discusses optimal dividends and bonus payments and the optimal control of pension funds.

The fourth provides a reconsideration of the problem of the minimization of the probability of ruin that as first introduced in Chapter 2. In particular, the author concentrates on the ruin probability of the risk process under optimal control and its asymptotic properties. Moreover, the author discusses the asymptotic behavior of the optimal controls. The main results are verified via a combination of the analysis of the Hamilton-Jacobi-Bellman equation and techniques from ruin theory. In this regard, the author only analyzes the Cramér-Lundberg case.

The appendix provides a terse overview of fundamental concepts related to stochastic processes, martingales, Markov processes, generators, change of measure techniques, risk theory, the Black-Scholes model and life insurance.

Reviewer: Mark A. Petersen (Potchefstroom)

MSC:

93-02	Research exposition (monographs, survey articles) pertaining to systems and control theory
60Kxx	Special processes
60G51	Processes with independent increments; Lévy processes
91B30	Risk theory, insurance (MSC2010)
93E03	Stochastic systems in control theory (general)

Keywords:

stochastic control; insurance