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On meromorphic functions sharing two one-point sets and two two-point sets. (English)

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The uniqueness theory of meromorphic functions mainly studies conditions under which there exists essentially only one function satisfying these conditions. There are many results about shared values, such as Nevanlinna's five values theorem, four values theorem, three values theorem and two values theorem. In this paper, the authors study two meromorphic functions sharing two one-points and two two-point sets CM, which extends the previous work.

But, in this paper, the cited Theorem A1 seems not to be perfectly right. In fact, Theorem A1 should be implied by the following results in [*R. Nevanlinna*, Acta Math., 48, 367–391 (1926; JFM 52.0323.03)] or [*C. C. Yang* and *H. X. Yi*, Uniqueness theory of meromorphic functions. Mathematics and its Applications (Dordrecht) 557. Dordrecht: Kluwer Academic Publishers. Beijing: Science Press. (2003; Zbl 1070.30011)]:

Theorem 1. Let f and g be distinct non-constant meromorphic functions and $a_i(j=1,2,3,4)$ be four distinct values. If f and g share $a_j(j=1,2,3,4)$ CM, then $f(z)=T(g(z),)$ where T is a Möbius transformation such that two of the four values are fixed points and another two (which are Picard exceptional values of f and g) exchange each other under T.

Theorem 2. Let f and g be distinct non-constant meromorphic functions sharing distinct values $a_i(j=1,2,3,4)$ CM. If $f(z)\not\equiv g(z)$, then $f(z)$ is a Möbius transformation of $g(z)$. Furthermore, two of $a_j(j=1,2,3,4)$, say a_1,a_2, are Picard exceptional values of $f(z)$ and $g(z)$, and the cross ratio $(a_1,a_2,a_3,a_4)=-1$.

Reviewer: Zhibo Huang (Guangzhou)

MSC:

30D35 Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

Keywords:

Uniquess theorem; shared values; Nevalinna theorem

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