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Bifurcations in the regularized Ericksen bar model. (English) Zbl 1134.74018
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Summary: We consider the regularized Ericksen model of an elastic bar on elastic foundation on an interval with Dirichlet boundary conditions as a two-parameter bifurcation problem. We explore, using local bifurcation analysis and continuation methods, the structure of bifurcations from double zero eigenvalues. Our results provide evidence in support of *S. Müller's* conjecture [Calc. Var. Partial Diff. Equ. 1, No. 2, 169–204 (1993; Zbl 0821.49015)] concerning the symmetry of local minimizers of the associated energy functional and describe in detail the structure of primary branch connections that occur in this problem. We give a reformulation of Müller conjecture and suggest two further conjectures based on the local analysis and numerical observations. We conclude by analysing a “loop” structure that characterizes $(k, 3k)$ bifurcations.

MSC:

74G60 Bifurcation and buckling

74K10 Rods (beams, columns, shafts, arches, rings, etc.)

Cited in 1 Document

Keywords:

Lyapunov-Schmidt analysis; continuation methods; Müller conjecture

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