Kurihara, Masato

On the structure of ideal class groups of CM-fields. (English) Zbl 1135.11339

Doc. Math. Extra Vol., Kazuya Kato's Fiftieth Birthday, 539-563 (2003).

In this interesting and well-written paper, the author proves a result on the structure of the p-part A_K of the ideal class group of the top field K in an abelian CM extension K/k, which generalises a result of Kolyvagin and Rubin in the absolutely abelian case. The main result of the present paper assumes that the odd prime p does not divide the order of the Galois group $G = G(K/k)$ (the so-called semisimple case) and the vanishing of the cyclotomic μ-invariant for the top field K. It gives a complete determination of the $\mathbb{Z}_p[G]$-module A_K up to isomorphism. In different words, it determines all the Fitting ideals of this module in terms of higher Stickelberger ideals. (The author has pointed out to the reviewer that there is a slight typo at the end of the statement of Theorem. 0.1: the last occurring Θ should have index $i-1$, not $i+1$.) The fact that a module is determined by the sequence of its Fitting ideals comes from the semisimplicity hypothesis.

The main argument uses the machinery of Euler and Kolyvagin systems. The really nice point of this is the following (which we oversimplify a little here): The Euler system of Gauss sums is not available when working over a general totally real base field. However, the theta (i.e. Stickelberger) elements attached to the usual system of abelian extensions F of K satisfy exactly the same relations. So if one takes suitable "preimages" g_F of these theta elements in the minus part of the multiplicative groups of all the F (which is made possible by the validity of Brumer's conjecture in this case), and if one throws away the cyclotomic character components, then these g_F are automatically unique, and they turn into a perfectly functional substitute for the Euler system of Gauss sums. This is a very natural idea, but the reviewer has not seen it being used in this profitable way before. The Kolyvagin system that arises from this Euler system is tightly linked with the higher Stickelberger elements (which are also defined by some kind of derivation process), and this, when fed into the machinery, allows one to prove the main result.

In an appendix, the author proves a result about the zeroth Fitting ideal of an Iwasawa module attached to an abelian extension F/k under certain hypotheses which do not include the restriction that p be coprime to $[F:k]$. (This result is not needed in the main part of the paper.) As the author mentions, there are related results of the reviewer (reference [4] in the paper, which now has appeared [Math. Z. 246, No. 4, 733–767 (2004; Zbl 1067.11067)]).

Reviewer: Cornelius Greither (Neubiberg) (MR2046607)

MSC:

11R29 Class numbers, class groups, discriminants
11R23 Iwasawa theory

Full Text: EuDML EMIS