Wang, Tongke

Alternating direction finite volume element methods for 2D parabolic partial differential equations. (English) Zbl 1135.65037

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The author combines finite volume methods and alternating direction methods for two dimensional parabolic equations. He adopts the ideas of *J. Douglas jun.* and co-authors [Numerical Solution partial diff. Equations II, Proc. 2nd Sympos. numerical Solution partial diff. Equations, SYNSPADE 1970, Univ. Maryland, 133–214 (1971; Zbl 0239.65088) and Math. Models Methods Appl. Sci. 11, No. 9, 1563–1579 (2001; Zbl 1012.65095)] and writes the finite volume element schemes as tensor product forms so that he can convert them to a series of one-dimensional problems, which can be solved alternatively.

He gives three kinds of alternating direction methods, the first two are similar to Douglas schemes [Zbl 0239.65088] and [Zbl 1012.65095] in the finite element method and the finite difference method, the third is an extension of the locally one-dimensional finite difference scheme [Zbl 1012.65095] with second order accuracy. He obtains optimal error estimates in L_2 or H^1 semi-norms for these schemes and illustrates that in two numerical examples.

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MSC:

65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
35K05 Heat equation
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

Keywords:

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