

Bourbaki, Nicolas

Elements of mathematics. Algebra. Chapters 4–7. Reprint of the 1981 original. (Éléments de mathématique. Algèbre. Chapitres 4 à 7.) (French) [Zbl 1139.12001](#)
Berlin: Springer (ISBN 3-540-34398-9/pbk). vii, 422 p. (2007).

Book II of N. Bourbaki’s fundamental and widely encyclopedic work “Éléments de Mathématique” is titled “Algebra” and encompasses (until now) ten chapters, which were successively published between 1943 and 1980, originally in French.

The volume under review is the faithful and unabridged reprinting of Chapters 4–7 of Bourbaki’s treatise on algebra, whose French original, in this arrangement, was first published in (1981; [Zbl 0498.12001](#)). Together with its masterly English translation by *P. M. Cohn* and *J. Howie* ([Zbl 0719.12001](#)), this volume has been reviewed repeatedly in the past, and therefore we may refer to these reviews in order to avoid any superfluous reiteration.

However, it should be recalled that Chapters 4–7 were successively written in the 1950s, that is, by the founders of the Bourbaki group themselves, thereby reflecting their former revolutionary spirit and style in its original fashion, and that Bourbaki’s “Algebra” has been among those parts of their monumental work that have become particularly pioneering, important, guiding and popular in view of the development of pure mathematics in the second half of the 20th century.

Also, it should be recollected that Chapters 4–7 of Bourbaki’s “Algebra” focus on a systematic, rigorous and comprehensive treatment of field theory, Galois theory, and the theory of modules over a principal domain, together with numerous related topics and applications.

More precisely, Chapter 4 develops the abstract theory of general polynomial rings, function fields, and formal power series, including the differential aspects (differentials and derivations) of these topics as a fundamental part. This is enriched by an in-depth treatment of symmetric tensor algebras, divided powers, polynomial maps, and their functorial interrelations, on the one hand, and by a just as comprehensive discussion of symmetric polynomials, symmetric rational functions, symmetric power series, resultants, and discriminants, on the other.

Chapter 5 is then devoted to the theory of commutative fields, their various kinds of extensions, and the allied theory of étale algebras over a ground field. Apart from the fundamentals of Galois theory, Kummer theory, Artin-Schreier theory, and of the theory of finite fields, this chapter also discusses separable algebras and differential criteria for separability in full generality and detail.

Chapter 6 briefly describes the basics of ordered groups and ordered fields, together with their respective fundamental structure theorems, whereas Chapter 7 deals with the theory of modules over a principal domain and its applications to the study of endomorphisms of finite-dimensional vector spaces.

Today, fifty years after the first appearance of these chapters of Bourbaki’s fundamental treatise on algebra, all their contents look pretty standard and can be found in various contemporary textbooks, too. Nevertheless, one has to keep in mind that this fact is due to the existence of Bourbaki’s pioneering classic, which still serves as the prototype for the (ultra-)modern approach to abstract algebra, and as an indispensable reference for its numerous successors as well. In particular, it is the vast supply of Bourbaki-style exercises accompanying the entire text, together with the precious historical remarks to each chapter, that makes Bourbaki’s text so absolutely unique and unsurpassed.

No doubt, also Bourbaki’s Chapters 4–7 of Book II will remain an everlasting treasure in the textbook literature of abstract algebra.

Reviewer: [Werner Kleinert \(Berlin\)](#)

MSC:

- 12-02 Research exposition (monographs, survey articles) pertaining to field theory
- 13-02 Research exposition (monographs, survey articles) pertaining to commutative algebra
- 00A05 Mathematics in general
- 01A72 Schools of mathematics
- 12Fxx Field extensions
- 13F10 Principal ideal rings
- 13C10 Projective and free modules and ideals in commutative rings
- 12E05 Polynomials in general fields (irreducibility, etc.)
- 06F15 Ordered groups
- 05E05 Symmetric functions and generalizations

Cited in **2** Reviews
Cited in **16** Documents

Keywords:

general mathematics; textbook; field extensions; Galois theory; polynomial rings; principal domains; modules; algebras; endomorphisms; ordered fields

Full Text: [DOI](#)