Field, Michael; **Melbourne, Ian**; **Török, Andrei**
Stability of mixing and rapid mixing for hyperbolic flows. (English) Zbl 1140.37004
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This work focuses in C^r-stability of mixing and on the rate of mixing for Axiom A and Anosov flows.

Let M be a compact connected manifold and let Φ_t be a C^1 flow on M. A Φ_t-invariant set Λ is (topologically) mixing if for any nonempty open sets $U, V \subset \Lambda$ there exists $T > 0$ such that $\Phi_t(U) \cap V \neq \emptyset$ for all $t > T$. The flow is stably mixing if all nearby flows (in the appropriate topology) are mixing.

Let $\mathcal{A}_r(M)$ denote the set of C^r flows ($1 \leq r \leq \infty$) on M satisfying the Axiom A and the no cycle property. It is well known that the nonwandering set Ω of such flows admits the spectral decomposition $\Omega = \Lambda_1 \cup \cdots \cup \Lambda_k$, where Λ_i are disjoint topologically transitive locally maximal hyperbolic sets. The sets Λ_i are called (hyperbolic) basic sets. A basic set is nontrivial if it is neither an equlibrium nor a periodic solution.

One of main results in this work is the following

Theorem 1.1.

(a) Suppose $2 \leq r \leq \infty$. There is a C^2-open, C^r-dense subset of flows in $\mathcal{A}_r(M)$ for which each nontrivial basic set is mixing.

(b) Suppose $1 \leq r \leq \infty$. There is a C^1-open, C^r-dense subset of flows in $\mathcal{A}_r(M)$ for which each nontrivial attracting basic set is mixing.

The other main result (which extends the first one) shows that typical Axiom A flows are stably rapid mixing. Suppose that Λ is a basic set for an Axiom A flow Φ_t and let μ be an equilibrium state for a Hölder potential. Given $A, B \in L^2(\Lambda, \mu)$, the correlation function is

$$\rho_{A,B}(t) = \int_\Lambda A \circ \Phi_t B \, d\mu - \int_\Lambda A \, d\mu \int_\Lambda B \, d\mu$$

The flow is mixing if and only if $\rho_{A,B}(t) \to 0$ as $t \to \infty$ for all $A, B \in L^2(\Lambda, \mu)$. The flow is rapid mixing if for any $n > 0$, there is a constant $C \geq 1$ such that

$$|\rho_{A,B}(t)| \leq C\|A\| \, \|B\| t^{-n}, \quad t > 0$$

for all observations A, B that are sufficiently smooth in the flow direction. The notion of rapid mixing is independent of the choice of the equilibrium state μ.

Theorem 1.6.

(a) Suppose $2 \leq r \leq \infty$. There is a C^2-open, C^r-dense subset of flows in $\mathcal{A}_r(M)$ for which each nontrivial basic set is rapid mixing.

(b) Suppose $1 \leq r \leq \infty$. There is a C^1-open, C^r-dense subset of flows in $\mathcal{A}_r(M)$ for which each nontrivial attracting basic set is rapid mixing.

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37D20 Uniformly hyperbolic systems (expanding, Anosov, Axiom A, etc.)

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