

**Calvaruso, G.; Perrone, D.**

***H*-contact unit tangent sphere bundles.** (English) Zbl 1140.53014

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Let  $(M, g)$  be a Riemannian manifold. Its unit tangent bundle  $T_1M$  has a natural contact metric structure  $(\xi, \eta, \Phi, \tilde{g})$ . Following a previous paper of the author [Differ. Geom. Appl. 20, No. 3, 367-378 (2004; Zbl 1061.53028)] we say that a metric contact structure is *H*-contact if the Reeb vector field  $\xi$  is harmonic. E. Boeckx and L. Vanhecke [Differ. Geom. Appl. 13, No. 1, 77-93 (2000; Zbl 0973.53053)] have proved that if  $M$  is a two-point homogeneous space then  $T_1M$  is *H*-contact, and asked if the converse of this holds. This paper investigates this question, confirming in several cases.

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#### MSC:

**53C15** General geometric structures on manifolds (almost complex, almost product structures, etc.) Cited in 8 Documents

**53C25** Special Riemannian manifolds (Einstein, Sasakian, etc.)

**53C35** Differential geometry of symmetric spaces

#### Keywords:

[contact metric manifold](#); [unit tangent sphere bundle](#); [H-contact spaces](#)

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