Li, Ren-Cang
Vandermonde matrices with Chebyshev nodes. (English) Zbl 1140.65025
Linear Algebra Appl. 428, No. 8-9, 1803-1832 (2008).

The paper is concerned with a study of Vandermonde matrices and so called rectangular Vandermonde matrices, whose elements (nodes) are the translated Chebyshev zero nodes, respectively, the translated Chebyshev extreme nodes (the translated zeros, respectively the translated extreme points of the Chebyshev polynomial of the first kind). Such matrices have many interesting properties derived from a simple QR or QR-like decomposition.

Two immediate applications of studying Vandermonde matrices with translated Chebyshev nodes are presented. The first one is to establish asymptotically optimal lower bounds on the condition numbers of real rectangular Vandermonde matrices and establishes nearly optimal condition on a given interval for such matrices. The second application is their implications to the convergence analysis of conjugate gradient method, of minimal residual method and of generalized minimal residual method. The author solves a few minimization problems, each related to residual of the methods mentioned above, on certain linear systems. For rectangular Vandermonde matrices, an exact expression for the Frobenius condition number is derived. The results, as the author says, can be extended to Vandermonde matrices with nodes being zeros of any translated orthogonal polynomial other than Chebyshev ones.

Reviewer: Iulian Coroian (Baia Mare)

MSC:
65F10 Iterative numerical methods for linear systems
65F35 Numerical computation of matrix norms, conditioning, scaling

Keywords:
Vandermonde matrix; Chebyshev polynomial; conjugate gradient method; minimal residual method; generalized minimal residual method; convergence; condition numbers

Software:
mctoolbox

Full Text: DOI

References:
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