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Sharp Lieb-Thirring inequalities in high dimensions. (English) Zbl 1142.35531
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From the introduction: Let us consider a Schrödinger operator in $L^2(\mathbb{R}^d)$, $-\Delta + V$, where V is a real-valued function. *E. H. Lieb* and *W. E. Thirring* [Inequalities for the moments of the eigenvalues of the Schrödinger Hamiltonian and their relation to Sobolev inequalities. *Stud. math. Phys., Essays Honor Valentine Bargmann*, 269–303 (1976; [Zbl 0342.35044](#))] proved that if $\gamma > \max(0, 1 - \frac{1}{2}d)$, then there exist universal constants $L_{\gamma,d}$ satisfying

$$\operatorname{tr}(-\Delta + V)_-^\gamma \leq L_{\gamma,d} \int_{\mathbb{R}^d} V_-^{\gamma+d/2}(x) dx.$$

The main purpose of this paper is to verify $L_{\gamma,d} = L_{\gamma,d}^{\text{cl}}$ for any $\gamma \geq \frac{3}{2}$, $d \in \mathbb{N}$ and any $V \in L^{\gamma+d/2}(\mathbb{R}^d)$.

MSC:

[35P15](#) Estimates of eigenvalues in context of PDEs
[35J10](#) Schrödinger operator, Schrödinger equation
[47F05](#) General theory of partial differential operators
[81U05](#) 2-body potential quantum scattering theory

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