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Generating families in a topos. (English) [Zbl 1143.18006](#)

[Theory Appl. Categ. 16, 896-922 \(2006\)](#).

A generating family in a category \mathcal{C} is a collection of objects $\{A_i | i \in I\}$ such that if for any subobject $Y \xrightarrow{m} X$, every $f : A_i \rightarrow X$ factors through m , then m is an isomorphism – i.e., the functors $\mathcal{C}(A_i, -)$ are collectively conservative.

In this paper, we examine some circumstances under which subobjects of 1 form a generating family. Objects for which subobjects of 1 do form a generating family are called partially well-pointed. For a Grothendieck topos, it is well known that subobjects of 1 form a generating family if and only if the topos is localic. For the elementary case, little more is known. The problem is studied by *F. Borceux* [*Cah. Topol. Géom. Différ.* 16, 3–15 (1975; [Zbl 0311.18006](#))], where it is shown that the result is internally true, an equivalent condition is found in the boolean case, and certain preservation properties are shown. We look at two different approaches to the problem, one based on a generalization of projectivity, and the other based on looking at the most extreme sorts of counterexamples.

MSC:

[18B25](#) Topoi

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