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On the Benson-Ratcliff invariant of coadjoint orbits on nilpotent Lie groups. (English)

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Let G be a connected Lie group with Lie algebra $\mathfrak{g}$ and $\mathfrak{g}^*$ the dual vector space of $\mathfrak{g}$. The cohomology of the complex $\wedge(\mathfrak{g}^*)$ is denoted by $H^*(\mathfrak{g})$. Let $\mathcal{O} \subset \mathfrak{g}^*$ be a coadjoint orbit of G with dimension $2q$. For any $\ell \in \mathcal{O}$, regarded as an element of $\wedge^1(\mathfrak{g}^*)$, the differential form $\ell \wedge (d\ell)^q$ is a closed form belonging to $\wedge^{2q+1}(\mathfrak{g}^*)$. *C. Benson* and *G. Ratcliff* [Mich. Math. J. 34, 23–30 (1987; Zbl 0618.22005)] proved that the cohomology class $[\ell \wedge (d\ell)^q] \in H^{2q+1}(\mathfrak{g})$ is independent of the choice of $\ell \in \mathcal{O}$.

When G is an exponential solvable Lie group, every irreducible unitary representation π of G is uniquely associated with a coadjoint orbit $\mathcal{O}_\pi$ via the Kirillov-Bernat mapping. Let us define

$$i(\pi) = i(\mathcal{O}_\pi) = [\ell \wedge (d\ell)^q] \in H^{2q+1}(\mathfrak{g}), \quad \ell \in \mathcal{O}_\pi.$$

In the paper cited above, *C. Benson* and *G. Ratcliff* presented the following conjecture. Let G be a connected and simply connected nilpotent Lie group with one-dimensional center. Let $\ell \in \mathfrak{g}^*$ be a linear form dual to a basis element of the center and π_ℓ the irreducible unitary representation of G corresponding to the coadjoint orbit $G \cdot \ell$. Then $i(\pi_\ell) \neq 0$.

In this paper the authors first give a counterexample to this conjecture, then they study some cases where the conjecture holds. They also try to separate irreducible unitary representations of G by means of slightly modified $i(\pi)$.

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MSC:

22E27	Representations of nilpotent and solvable Lie groups (special orbital integrals, non-type I representations, etc.)	Cited in **1** Document
22D10	Unitary representations of locally compact groups	

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nilpotent Lie group; Lie algebra cohomology; orbit method