Jeanjean, Louis; **Tanaka, Kazunaga**
A positive solution for a nonlinear Schrödinger equation on $\mathbb{R}^N$. (English) Zbl 1143.35321
Indiana Univ. Math. J. 54, No. 2, 443-464 (2005).

The authors are concerned with the existence of a positive solution $u \in H^1(\mathbb{R}^N)$ of the non-autonomous Schrödinger equation $-\Delta u + V(x)u = f(u)$ in $\mathbb{R}^N$, $N \geq 2$, where $f : [0, +\infty) \to \mathbb{R}$ and $V : \mathbb{R}^N \to \mathbb{R}$ are continuous functions. The main feature of this paper is that the authors replace the standard global Ambrosetti-Rabinowitz superlinear condition $0 < \mu \int_0^t f(s)\, ds \leq t f(t)\, dt$, for all $t \in \mathbb{R}$ and some $\mu > 2$, with a less restrictive assumption on f, which does not need global conditions on the nonlinearity. More precisely, the nonlinear term f is supposed to have a superlinear and subcritical growth at infinity, while around the origin f satisfies $f(0) = 0$ and $f'(0)$ is defined. Under some standard assumptions on the variable potential V, the authors establish the existence of at least one nontrivial positive solution. The proof of this result relies on appropriate variational methods, in which the "problem at infinity" plays an important role. This approach is crucial to ensure the compactness of bounded Palais-Smale sequences. The method developed in the present paper can be generalized for solving other classes of nonlinear stationary differential equations lacking compactness.

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MSC:

35J60	Nonlinear elliptic equations
58E05	Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
35B09	Positive solutions to PDEs
35J20	Variational methods for second-order elliptic equations
47J30	Variational methods involving nonlinear operators

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