The monograph represents results on stability and attractivity obtained recently for continuous-time dynamical systems formulated as measure differential inclusions. The notion of the differential inclusion arises from differential equations with discontinuous right-hand side \( \dot{x}(t) = f(t, x(t)) \) and allows to give a generalized definition of solution for this of type equations. This generalization can be obtained with use of Filippov’s convex method. It consists in filling in the graph of \( f(t, x(t)) \) with a convex set, that results in a differential inclusion

\[
\dot{x}(t) \in \mathcal{F}(t, x(t))
\] (1)

with set-valued \( \mathcal{F}(t, x) \). Solutions of (1) are absolutely continuous functions. In order to describe discontinuities (jumps) in the state \( x(t) \) at isolated time-instances, J. J. Moreau proposed an extension of (1) to the measure differential inclusion

\[
dx \in d\Gamma(t, x(t)),
\] (2)

where \( d\Gamma(t, x(t)) \) is a set-valued measure function consisting of Lebesgue integrable part and an atomic part, i.e.

\[
d\Gamma(t, x(t)) = \mathcal{F}(t, x(t)) + \mathcal{G}(t, x(t)).
\]

The measure differential inclusion (2) has to be understood in the sense of integration. Solutions of (2) are functions of locally bounded variation in time. Set-valued force laws can be used to describe contact forces for unilateral constraints with friction and impact.

Non-smooth potential theory is developed in the book for set-valued force laws. The set-valued forces are incorporated in Newton-Euler equations of motion as Lagrange multipliers. It results in measure differential inclusions describing mechanical systems with unilateral constraints. Definitions of stability and attractivity properties are given for equilibria and positively invariant sets of time-autonomous measure differential inclusions and for solutions \( x(t), t \in (t_*, +\infty) \), of non-autonomous measure differential inclusions. Basic Lyapunov stability theorems are generalized to equilibrium points and positively invariant sets of autonomous measure differential inclusions. Generalizations of LaSalle’s invariance principle and of Chetaev’s instability theorem are given for the time-autonomous case, too.

These results are applied to Lagrangian mechanical systems with frictional unilateral and bilateral constraints. When studying this class of systems, it is natural to take a Lyapunov function equal to the total mechanical energy, that includes the potential of normal contact forces of unilateral constraints. It allows to obtain Lyapunov-type theorems for stability, attractivity and instability of equilibrium positions and sets. The use of these theorems is demonstrated for a number of mechanical systems (falling block, rocking bar, constrained bar). At last, theorems are proved which give sufficient conditions for the uniform convergence of measure differential inclusions with certain maximal monotonicity properties. Following B. P. Demidovich, the system (2) is said to be convergent (uniformly convergent) if there exists a unique solution \( \pi(t) \) that is bounded on the whole time axis and this solution is globally attractively stable (globally uniformly attractively stable). The convergence property is highly instrumental in the control theory for solving the problems of tracking, synchronization and observer design. Some illustrative examples of convergent mechanical systems with set-valued force laws are presented.

The monograph will be of interest to researchers and engineers working in the field of non-smooth dy-
namic and mechanics.

Reviewer: Boris Ivanovich Konosevich (Donetsk)

**MSC:**

- **70-02** Research exposition (monographs, survey articles) pertaining to mechanics of particles and systems
- **70K20** Stability for nonlinear problems in mechanics
- **70F40** Problems involving a system of particles with friction

**Keywords:**

non-smooth mechanics; set-valued function; measure differential inclusion; Lyapunov stability

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