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Summary: When two or more branches of a function merge, the Chebyshev series of $u(\lambda)$ will converge very poorly with coefficients $a_n$ of $T_n(\lambda)$ falling as $O(1/n^\alpha)$ for some small positive exponent $\alpha$. However, as shown in [J.P. Boyd, Appl. Math. Comput. 143, No. 2–3, 189–200 (2003; Zbl 1025.65042)], it is possible to obtain approximations that converge exponentially fast in $n$. If the roots that merge are denoted as $u_1(\lambda)$ and $u_2(\lambda)$, then both branches can be written without approximation as the roots of $(u - u_1(\lambda))(u - u_2(\lambda)) = u^2 + \beta(\lambda)u + \gamma(\lambda)$. By expanding the nonsingular coefficients of the quadratic, $\beta(\lambda)$ and $\gamma(\lambda)$, as Chebyshev series and then applying the usual roots-of-a-quadratic formula, we can approximate both branches simultaneously with error that decreases proportional to $\exp(-\sigma N)$ for some constant $\sigma > 0$ where $N$ is the truncation of the Chebyshev series. This is dubbed the “Chebyshev-Shafer” or “Chebyshev-Hermite-Padé” method because it substitutes Chebyshev series for power series in the generalized Padé approximants known variously as “Shafer” or “Hermite-Padé” approximants. Here we extend these ideas. First, we explore square roots with branches that are both real-valued and complex-valued in the domain of interest, illustrated by meteorological baroclinic instability. Second, we illustrate triply branched functions via roots of the Kepler equation, $f(u; \lambda, \epsilon) \equiv u - \epsilon \sin(u) - \lambda = 0$. Only one of the merging roots is real-valued and the root depends on two parameters ($\lambda, \epsilon$) rather than one. Nonetheless, the Chebyshev-Hermite-Padé scheme is successful over the whole two-dimensional parameter plane. We also discuss how to cope with poles and logarithmic singularities that arise in our examples at the extremes of the expansion domain.

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