Bundschuh, Peter; **Zudilin, Wadim**

Irrationality measures for certain q-mathematical constants. (English) Zbl 1153.11034

Math. Scand. 101, No. 1, 104-122 (2007).

For $q \in \mathbb{C}$ with $|q| > 1$ define

$$\pi_q = 1 + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{q^{2n-1} - 1} \cdot p$$

S. Chowla [Proc. Nat. Inst. Sci. India, Part A 13, 171–173 (1947; Zbl 1153.11318)] and *P. Erdős* [J. Indian Math. Soc., II. Ser. 12, 63–66 (1948; Zbl 0032.01701)] showed that π_q is irrational when $q \in \mathbb{Z} \setminus \{0, 1, -1\}$. The authors ["Rational approximations to a q-analogue of π and some other q-series", Diophantine approximation. Festschrift for Wolfgang Schmidt. Wien: Springer, Dev. Math. 16, 123–139 (2008; Zbl 1213.11146)] gave an irrationality measure for these π_q: they proved with $\mu = 10.317\ldots$ that the inequality

$$\left| \pi_q - \frac{P}{Q} \right| < Q^{-\mu}$$

has only finitely many solutions in rational numbers P/Q. Here they refine their estimate and prove the same result with the sharper value $\mu = 6.503\ldots$ They deduce similar results for the numbers

$$\lambda_q = \sum_{n=1}^{\infty} \frac{1}{q^{2n-1} - 1} \quad \text{and} \quad \beta_q = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{q^n + 1}$$

with the exponents $3.898\ldots$ and $3.940\ldots$ respectively.

Reviewer: Michel Waldschmidt (Paris)

MSC:

11J82	Measures of irrationality and of transcendence
11M36	Selberg zeta functions and regularized determinants; applications to spectral theory, Dirichlet series, Eisenstein series, etc. (explicit formulas)
30B50	Dirichlet series, exponential series and other series in one complex variable

Keywords:

Lambert series; irrationality measure; rational approximation

Full Text: DOI