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Summary: We show that for convex domains in Euclidean space, Cheeger’s isoperimetric inequality, spectral-gap of the Neumann Laplacian, exponential concentration of Lipschitz functions, and the a priori weakest uniform tail-decay of these functions, are all equivalent (to within universal constants, independent of the dimension). This substantially extends previous results of Maz’ya, Cheeger, Gromov-Milman, Buser and Ledoux. As an application, we conclude the stability of the spectral-gap for convex domains under convex perturbations which preserve volume (up to constants) and under maps which are “on-average” Lipschitz. We also provide a new characterization of the Cheeger constant, as one over the expectation of the distance from the “worst” Borel set having half the measure of the convex domain. In addition, we easily recover (and extend) many previously known lower bounds, due to Payne-Weinberger, Li-Yau and Kannan-Lovász-Simonovits, on the Cheeger constant of convex domains. Essential to our proof is a result from Riemannian geometry on the concavity of the isoperimetric profile. Our results extend to the more general setting of Riemannian manifolds with density which satisfy the $CD(0,\infty)$ curvature-dimension condition of Bakry-Émery.

MSC:

49Q15 Geometric measure and integration theory, integral and normal currents in optimization
26A16 Lipschitz (Hölder) classes
52B60 Isoperimetric problems for polytopes

Keywords:

Cheeger’s isoperimetric inequality; spectral-gap of the Neumann Laplacian; exponential concentration of Lipschitz functions; Riemannian manifolds

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