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Optimal surrender strategies for equity-indexed annuity investors. (English) Zbl 1156.91379

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Summary: An equity-indexed annuity (EIA) is a hybrid between a variable and a fixed annuity that allows the investor to participate in the stock market, and earn at least a minimum interest rate. The investor sacrifices some of the upside potential for the downside protection of the minimum guarantee. Because EIAs allow investors to participate in equity growth without the downside risk, their popularity has grown rapidly.

An optimistic EIA owner might consider surrendering an EIA contract, paying a surrender charge, and investing the proceeds directly in the index to earn the full (versus reduced) index growth, while using a risk-free account for downside protection. Because of the popularity of these products, it is important for individuals and insurers to understand the optimal policyholder behavior.

We consider an EIA investor who seeks the surrender strategy and post-surrender asset allocation strategy that maximizes the expected discounted utility of bequest. We formulate a variational inequality and a Hamilton-Jacobi-Bellman equation that govern the optimal surrender strategy and post-surrender asset allocation strategy, respectively. We examine the optimal strategies and how they are affected by the product features, model parameters, and mortality assumptions. We observe that in many cases, the "no-surrender" region is an interval (w_l, w_u); i.e., that there are two free boundaries. In these cases, the investor surrenders the EIA contract if the fund value becomes too high or too low. In other cases, there is only one free boundary; the lower (or upper) surrender threshold vanishes. In these cases, the investor holds the EIA, regardless of how low (or high) the fund value goes. For a special case, we prove a succinct and intuitive condition on the model parameters that dictates whether one or two free boundaries exist.

MSC:

91B28	Finance etc. (MSC2000)
60G40	Stopping times; optimal stopping problems; gambling theory
60H30	Applications of stochastic analysis (to PDEs, etc.)
91B30	Risk theory, insurance (MSC2010)

Keywords:

optimal investment; optimal stopping; free boundary problem; equity-indexed annuity

Full Text: DOI Link

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