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Apéry limits of differential equations of order 4 and 5. (English) Zbl 1159.14019

Yui, Noriko (ed.) et al., Modular forms and string duality. Proceedings of a workshop, Banff, Canada, June 3–8, 2006. Providence, RI: American Mathematical Society (AMS); Toronto: The Fields Institute for Research in Mathematical Sciences (ISBN 978-0-8218-4484-7/hbk). Fields Institute Communications 54, 105-123 (2008).

In 1978, Apéry proved the irrationality of $\zeta(3)$ by realizing $\zeta(3)$ as the limit of some third order recursion. The so-called Apéry limits were then computed for several other recursions of higher orders. The formal power series constructed using the coefficients of the recursions turn out to be related to solutions of differential equations. The paper under review extends the concept of Apéry limits to higher order differential equations.

In fact, it presents more examples of differential equations of order 4 and 5 of Calabi–Yau type, and compute the Apéry limits. For those equations obtained from Hadamard products of second and third order equations, it is proved that the Apéry limits are determined by the factors. For other equations, based on empirical evidence, it is conjectured that the Apéry limits are rational linear combinations of certain transcendental numbers, and special values of L -series.

For the entire collection see [[Zbl 1147.11005](#)].

Reviewer: [Noriko Yui \(Kingston\)](#)

MSC:

- [14J32](#) Calabi-Yau manifolds (algebraic-geometric aspects)
- [11Y60](#) Evaluation of number-theoretic constants
- [14D05](#) Structure of families (Picard-Lefschetz, monodromy, etc.)

Cited in **2** Documents

Keywords:

[Apéry limit](#); [Calabi-Yau differential equation](#); [Hadamard product](#)