Steeb, Willi-Hans

Continuous symmetries, Lie algebras, differential equations, and computer algebra. 2nd ed. (English) [Zbl 1159.35001]

Lie symmetry theory is one of the most important approaches to the analysis of differential equations. In particular, in the context of mathematical physics symmetries are of great importance. For this reason it is not surprising that the number of textbooks devoted to symmetry theory has considerably increased in recent years (including two previous books by the same author).

The book can be roughly divided into three parts. The first nine chapters introduce the required mathematical tools: groups as abstract structure and as transformation groups, infinitesimal generators, Lie groups and algebras, vector fields and differential forms together with the basic operations on them. The next four chapters cover elementary symmetry theory. Here one can find what symmetries of a differential equation are (including generalised or Lie-Bäcklund symmetries) and how they can be determined. The question of symmetry reduction and the explicit integration of differential equations is treated only very briefly. Instead, a whole chapter is devoted to the inverse problem of constructing a differential equation with a prescribed symmetry algebra. Furthermore, a list of the symmetries of some common differential equations is given. The third and largest part consists of a variety of more advanced topics, in particular a number of applications of symmetry theory in the context of integrable systems and soliton equations. Covered are recursion operators, Bäcklund transformations, Lax pairs, conservation laws, Painlevé theory, Ziglin’s theorem, gauge theories, Bose operators and discrete dynamical systems.

A particular feature of the book is the strong emphasis on computer algebra. Each chapter ends with a section called Computer Algebra Applications. These sections usually describe how some typical computation connected with the corresponding chapter can be implemented in SymbolicC++, an extension of the C++ programming language for symbolic computations (rather strangely, it is nowhere mentioned that SymbolicC++ has been developed in the group of the author and his book on SymbolicC++ is not cited). A short introduction to SymbolicC++ is given in the last chapter and all required files can be downloaded from the author’s home page. However, all presented computations are of a fairly simple nature and actually the use of an interactive computer algebra system would probably have been better. More complex operations like the setting up and solution of the determining equations for the Lie symmetries of a differential equation are not discussed from an algorithmic point of view.

The style of writing is encyclopaedic and example based. While an impressive range of topics and examples is covered, the treatment of the individual topics is usually rather superficial. Furthermore, often concepts are used without a proper introduction. The emphasis lies mainly on the required computations which are often presented in great detail, whereas the underlying ideas are only briefly discussed (if at all). As a typical example one may consider the section on pseudo-spherical surfaces. Here, out of the blue, a concrete metric tensor is given. Then without explanations its Riemann curvature is computed, although the concept of curvature has nowhere appeared before. Finally, without further discussion it is shown that the sine-Gordon equation arises, if one requires that the curvature is $-2$. For a deeper understanding of symmetry theory a novice will probably have to resort to other textbooks like the classical one by P. J. Olver [Applications of Lie Groups to Differential Equations, Springer-Verlag (1986; Zbl 0588.22001)]. However, the wealth of examples and the detailed presentation of the required computations makes the book a useful complement to any more theoretically oriented introduction into symmetry theory.

Reviewer: Werner M. Seiler (Kassel)
MSC:

35-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to partial differential equations
22E60 Lie algebras of Lie groups
22E70 Applications of Lie groups to the sciences; explicit representations
35A30 Geometric theory, characteristics, transformations in context of PDEs
35Q51 Soliton equations
37K05 Hamiltonian structures, symmetries, variational principles, conservation laws (MSC2010)
37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
37K35 Lie-Bäcklund and other transformations for infinite-dimensional Hamiltonian and Lagrangian systems
68W30 Symbolic computation and algebraic computation
58J70 Invariance and symmetry properties for PDEs on manifolds

Keywords:
partial differential equation; Lie group; Lie algebra; Lie symmetry; computer algebra; Bäcklund transformation; recursion operator; Lax pair; conservation law; integrability; soliton equation; Painlevé test; Bose operator; similarity solution; SymbolicC++

Software:
MACSYMA; SymbolicC++