Mammitzsch, Volker
Optimal kernels. (English) Zbl 1159.62020
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Summary: Kernel functions $K(x)$ are widely used for smoothing purposes in statistics, for instance see *B. W. Silverman* [Density estimation for statistics and data analysis. Chapman and Hall (1986; Zbl 0617.62042)], or *M. P. Wand* and *M. C. Jones* [Kernel smoothing. Chapman and Hall (1995; Zbl 0854.62043)], and also *M. G. Schimek* [Glättungsverfahren in der Biometrie: ein historischer Abriss. Electron. J. GMS, Med. Inform. Biom. Epidemiol (2005)]. Usually, they have compact support and are of order (ν, k), which means that the j-th moment M_j is zero for $j < k$ except $j = \nu < k$ and is standardized appropriately for $j = \nu$. Here, M_j means the integral over the product of $K(x)$ and x^j. In the case of $\nu = 0$, K is called a standard kernel. Kernels of order (ν, k) are called optimal if they change the sign exactly $k-2$ times and minimize the asymptotical integrated mean square error. In the case of $k-\nu$ being even, *T. Gasser* et al. [J. R. Stat. Soc., Ser. B 47, 238–252 (1985; Zbl 0574.62042)] have constructed polynomials $K(x)$ of degree k with $K(-1) = 0 = K(+1)$ which restricted to $[-1, 1]$ have exactly $k-2$ sign changes and are of order (ν, k). In some special cases those K could be proved to be optimal. Later on *B. L. Granovsky* and *H.-G. Müller* [Int. Stat. Rev. 59, No. 3, 373–388 (1991; Zbl 0749.62024)] showed that optimal kernels are continuous functions with $K(-1) = 0 = K(+1)$ and are polynomials on their support. Unfortunately, the converse is true only in the case $k-\nu < 4$. *H.-G. Pfeifer* [Zur Theorie der optimalen Kernschätzer unter Momentenbedingungen. Dept. Math., Univ. Marburg (1991)] in his diploma thesis constructed polynomials $p_i(x)$ of degree k and reals α_i in $[0, 1]$ with $\alpha_1 < \cdots < \alpha_m$ such that the restriction K_i of p_i to $[-1, \alpha_i]$ is of order (ν, k) and fulfills the boundary condition $p_i(-1) = 0 = p_i(\alpha_i)$, $i = 1, \ldots, m$, where m is the integer part of $(k-\nu)/2$.

We prove a long-standing conjecture in its most general form, which in the standard case has been verified in one of our earlier papers [Stat. Decis. 19, No. 1, 1–8 (2001; Zbl 1159.62306)]. By means of the theory of Gegenbauer (ultraspherical) polynomials we show that in the general case of arbitrary k, ν with $0 \le \nu \le k-2$ the kernel K_m is optimal and give its explicit form.

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